Entropic Estimator in a General Supply Chain

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1 Introduction

This paper describes an estimation procedure based on a two-stage schema. The procedure is suitably customized for problems in the areas of enterprise dynamics and of the supply chain management [SLKSL00]. The advantage of the proposed estimation method is that it provides a way for an automatic construction of the generic estimation models.

In a previous paper [WKR01], the model for the representation of the enterprise dynamics was proposed. The present paper is concerned with the formulation and solution of the following estimation and forecasting problem. Given an enterprise model of the market with a set of nominal parameters and given some time-stamped sensory data with a level of uncertainty characterized by an entropy fluctuation tensor [KP98], generate a state estimate/forecast of the flow demanded by the market. The estimate/forecast is generated by a recursive two-stage schema in which the estimates are a perturbation of parameters.

We will establish the relation of this forecasting schema with standard filtering algorithms via Fisher information transformation [Fri98].

2 Model

We model a supply chain as a membrane network that operates on flows of goods and information [WKR01]. That is an operation dynamics of an element of a supply chain is encoded as rules of the following form

$$a_1^j\xi_1 \circledast a_2^j\xi_2 \circledast \ldots \circledast a_N^j\xi_N \xrightarrow{k_j} b_1^j\xi_1 \circledast b_2^j\xi_2 \circledast \ldots \circledast b_N^j\xi_N,\ j = 1,\ldots,r, \quad (1)$$

where $\xi_1, \ldots, \xi_N$ are the labels of the goods , services, or money flowing through the element, a_i^j, b_i^j, $i = 1, ..., N$, $j = 1, ..., r$ are input and output stoichiometric coefficients respectively representing the number of units of each item that enter in the j-th step of the process, and the flow rate constants, k_j, $j = 1, \ldots, r$, are parameters that characterize flow and storage capacity constraints of the underlined process.

A procedure described in [WKR01] produces a dynamic model of the form

$$\frac{d}{dt}\eta(t,\mathbf{k}) = \bar{\mathbf{A}}(\mathbf{k})\eta(t,\mathbf{k}) + \bar{\mathbf{B}}(\mathbf{k})\bar{\mathbf{u}}(t). \quad (2)$$

where matrices $\mathbf{A}$ and $\mathbf{B}$ depend on the rate coefficients $\mathbf{k} = (k_1, ..., k_r)$. We assume that the rate coefficients do not change with time, i.e.

$$\dot{\mathbf{k}} = \mathbf{0}. \quad (3)$$

Usually rate coefficients $\{k_j\}$ are not known exactly and determined by experimentation. We are going to setup a schema for estimation of these model parameters. First we differentiate (2) with respect to k_j and then interchange the order of differentiation (assuming sufficient continuity conditions) to get

$$\frac{d}{dt}\psi_j(t,\mathbf{k}) = \bar{\mathbf{A}}(\mathbf{k})\,|_{\bar{\mathbf{k}}}\psi_j(t,\mathbf{k}) + \frac{\partial\bar{\mathbf{A}}(\mathbf{k})}{\partial k_j}|_{\bar{\mathbf{k}}}\eta(t,\mathbf{k}) + \frac{\partial\bar{\mathbf{B}}(\mathbf{k})}{\partial k_j}|_{\bar{\mathbf{k}}}\bar{\mathbf{u}}(t) + \mathbf{G}\mathbf{w}(t), \quad (4)$$

where $\psi_j(t,\mathbf{k}) := \frac{\partial\eta(\mathbf{t},\mathbf{k})}{\partial k_j}$, $\bar{\mathbf{k}}$ is the mean value of parameters $\mathbf{k}$, and a "noise term" $\mathbf{G}\mathbf{w}(t)$ is added in order to take into account the fact that we do not know exactly the rate coefficients $\bar{\mathbf{k}}$. In this term, $\mathbf{w}(t)$ is a martingale process with zero mean and covariance equal to the inverse of the information matrix $\mathbf{\Lambda}$ that measures the knowledge uncertainty on these rate coefficients.

3 Optimal Forecast Engine

A control cluster depicted in Figure 1 operates as follows. Sensory data from the system under control (Enterprise Node) and from the Network is fed to the Estimator. The Estimator outputs an estimate of a state. Adapter uses this estimate of a system state in order to compute an estimate of model parameters. These estimates are used for updating the model. Node simulator produces next state using a model with currently available parameters. The difference

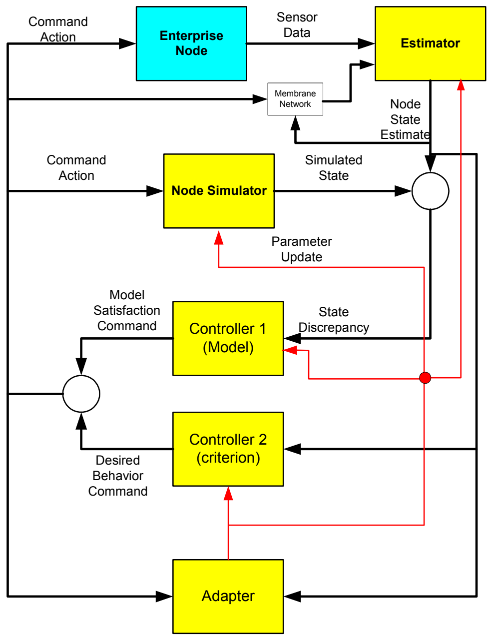


Figure 1: Block Diagram of a Control Cluster

between this simulated state and a new estimate of a system state is input into Controller1. A function of Controller 1 is to compute a control law that compensate for the model discrepancy. Controller 2 uses the estimate of a system state in order to compute an optimal control law that minimizes a certain criterion. In the present paper we will describe the Adapter.

We assume that an estimate of the incremental state of the system $\hat{\eta}$ can be computed. Then from (4), a system dynamics is described by the following stochastic differential equation

$$\mathbf{dx}(t) = \mathbf{A}\mathbf{x}(t)dt + \mathbf{B}\mathbf{u}(t)dt + \mathbf{G}\mathbf{dw}(t), \tag{5}$$

where $\mathbf{x}(t) := \begin{bmatrix} \psi_j(t,\mathbf{k}) \\ \hat{\eta} \end{bmatrix}$, $\mathbf{A} := \begin{bmatrix} \bar{\mathbf{A}}(\mathbf{k}) \mid_{\bar{\mathbf{k}}} \\ \frac{\partial \bar{\mathbf{A}}(\mathbf{k})}{\partial k_j} \mid_{\bar{\mathbf{k}}} \end{bmatrix}$, and $\mathbf{B} := \left[\frac{\partial \bar{\mathbf{B}}(\mathbf{k})}{\partial k_j} \mid_{\bar{\mathbf{k}}} \right]$.

The observation model compatible with the supply chain application is of the form

$$\mathbf{z}(t_n) = \mathbf{H}\mathbf{x}(t_n) + \mathbf{v}(t_n), \tag{6}$$

where the measurements are taken at discrete time instances t_n, $n = 1, 2, ...,$ not necessarily equally spaced and $\mathbf{v}(t_n)$ is a zero mean discrete martingale process with covariance $\mathbf{R}$ and independent of $\mathbf{w}(t)$ that models the level of uncertainty in the demand observations.

3.0.1 Time update between measurements

The conditional mean $\hat{x}(\tau|t_n)$ and covariance matrix $\Sigma(\tau|t_n)$, where $t_n \leq \tau < t_{n+1}$, are propagated according to the following equations

$$\hat{x}(\tau|t_n) = F\hat{x}(\tau|t_n) + Bu(\tau) \tag{7}$$

$$\dot{\Sigma}(\tau|t_n) = F\Sigma(\tau|t_n) + \Sigma(\tau|t_n)F^T + GQG^T \tag{8}$$

3.0.2 Measurement update

There is a discontinuity in trajectory $\hat{x}$ at times t_n. The following equations give the way of computing conditional mean $\hat{x}(t_{n+1}|t_{n+1})$ and covariance $\Sigma(t_{n+1}|t_{n+1})$ after the observation at time t_{n+1} was made

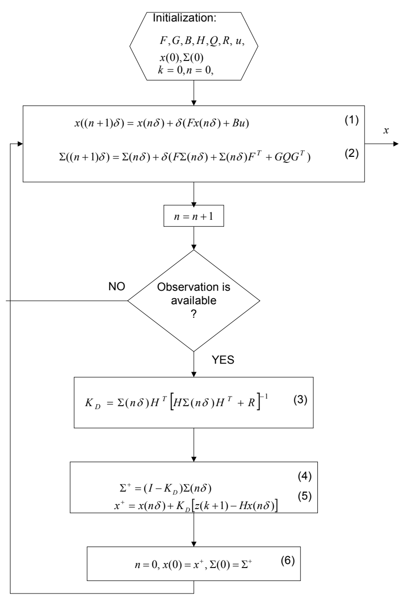
$$\hat{x}(t_{n+1}|t_{n+1}) = \hat{x}(t_{n+1}|t_n) + K_D(z(t_n)) - H\hat{x}(t_n|t_n)) \tag{9}$$

$$\Sigma(t_{n+1}|t_{n+1}) = [I - K_D H]\,\Sigma(t_{n+1}|t_n), \tag{10}$$

where (discrete) Kalman filter K_D is given as follows

$$K_D = \Sigma(t_{n+1}|t_n)H^T\left[H\Sigma(t_{n+1}|t_n)H^T + R\right]^{-1} \tag{11}$$

3.1 Algorithm



Flow Chart of the Algorithm

4 Connection to Fisher Information

In order to get estimates of $\mathbf{k}$ from $\hat{\mathbf{x}}(t)$ and $\hat{\eta}(t, \mathbf{k})$, consider a perturbation in $\hat{\eta}$

$$\Delta\hat{\eta}(t, \mathbf{k}) = \mathbf{\Psi}^T(t)\mathbf{\Delta k} + \dot{\hat{\eta}}(t, \mathbf{k})\Delta t, \tag{12}$$

where matrix $\mathbf{\Psi}^T(t) := \begin{bmatrix} \hat{\psi}_1(t) & \cdots & \hat{\psi}_r(t) \end{bmatrix}$ with entries obtained from $\hat{\mathbf{x}}(t)$.

From (12),

$$\mathbf{y}(t) = \mathbf{\Psi}^T(t)\mathbf{\Delta k}(t),$$

where $\mathbf{y}(t) := \Delta\hat{\eta}(t) - \bar{\mathbf{A}}\hat{\eta}(t) - \bar{\mathbf{B}}\bar{\mathbf{u}}(t)$ with matrices $\bar{\mathbf{A}}$ and $\bar{\mathbf{B}}$ evaluated at the current estimate of $\mathbf{k}$.

Then, by the continuous least-square estimation theory [AW95], the equations of the estimates that minimize lost function
$V(k) = \int\limits_0^t e^{-\alpha(t-\tau)} \left(y(\tau) - \mathbf{\Psi}^T(\tau)\mathbf{\Delta k}\right) d\tau$ are as follows:

$$\begin{aligned} \frac{d\,\widehat{\mathbf{\Delta k}}}{dt} &= \mathbf{P}(t)\mathbf{\Psi}(t)\mathbf{e}(t) \\ \mathbf{e}(t) &= y(t) - \mathbf{\Psi}^T(t)\widehat{\mathbf{\Delta k}} \\ \frac{d\mathbf{P}(t)}{dt} &= \alpha\mathbf{P}(t) - \mathbf{P}(t)\mathbf{\Psi}(t)\mathbf{\Psi}^T(t)\mathbf{P}(t), \end{aligned}$$

where, by definition, $\mathbf{P}^{-1}(t) := \mathbf{R}(t) := \int\limits_0^t e^{-\alpha(t-\tau)}\ \mathbf{\Psi}(\tau)\mathbf{\Psi}^T(\tau)d\tau$.

Note that matrix $\mathbf{R}(t)$, called Fisher information matrix, satisfies the following dynamics

$$\frac{d\mathbf{R}(t)}{dt} = -\alpha\mathbf{R}(t) + \mathbf{\Psi}(t)\mathbf{\Psi}^T(t).$$

5 Conclusions

In the present paper, we described a two-stage schema for estimation of the process rate parameters that appear in the dynamic model of a general supply chain. In the first stage, we estimate the parameter dependency functions $\psi_j(t, \mathbf{k})$. These estimates used as an input for the second stage, where a perturbation of parameters, $\mathbf{\Delta k}$, is estimated. We presented an algorithm for computing estimates of $\psi_j(t, \mathbf{k})$ in the case when observations of the system state are performed at discrete instances of time. Finally, we showed the connection to the Fisher information matrix.

In a future paper, we will present an approach of using this Fisher information matrix as a Lagrangian for the estimation problem. Then the estimates are quasi-geodetic trajectories in the Finsler space [DBS00] whose metric function is determined by the derived entropic tensor. This entropic tensor is obtained recursively on-line by an inverse Lagrangian schema [WKB]. The entropy tensor is a measure of uncertainty of the estimate/forecast and each geodetic segment trajectory is the time-estimate series. These segments are approximated by solutions of a variational optimization problem. We will establish the relation of this forecasting schema with standard filtering algorithms via Fisher information transformation [Fri98]. The domain of our estimation algorithm is in the tangent bundle of a suitably defined Finsler space and the algorithm is based on an approximation to the market model based on a spray vector residing in the tangent to a tangent bundle [Lan95]. This approach will be illustrated with a real-life example.

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