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Research Needs in Economic Analysis of Control Design Projects

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Abstract

We outline a new methodology for generating economic plans and budgets for design projects. This approach is based on an extension of the Kohn-Nerode hybrid system models and algorithms. Given a hybrid system model of a distributed system and a performance specification for that system (expressed as a relaxed variational problem of minimizing the integral of a Lagrangian over a trajectory on a suitable manifold), we extract a nearly optimal chattering control program which forces the system to obey its performance specification.

To adapt the Kohn-Nerode methodology to the problem of generating economic plans and budgets for design projects, we model each design project as the problem of finding a winning strategy for a suitable associated heterogeneous differential game. The game formulation captures in the game rules all interaction mechanisms between players (decision agents) participating in the design. This differential game formulation is a key step toward rendering distributed economic and planning problems in a form to which the Kohn-Nerode methods can be applied. We then rewrite the game rules as the definition of an associated "carrier manifold" and we restate the winning condition as a relaxed calculus of variations problem whose solution is a collection of winning strategies for the players. These relaxed variational strategies can then be approximated by chattering strategies for each player in accord with the Kohn-Nerode method. The Multiple Agent Hybrid Control Architecture (MAHCA) that implements the Kohn-Nerode models and algorithms is well suited, after small adjustments, for extracting nearly optimal strategies for the players in economic and planning games.

1. Introduction

Overcoming the limitations of traditional methods for finding optimal economic policies is a major challenge. This paper presents an approach which first reformulates the policy problem as one of solving a heterogeneous differential game and then solves the game by applying a variation of the Kohn-Nerode methodology for extracting control programs to find nearly optimal economic policies. The framework is applicable to many areas of economic policy modeling. The Multiple Agent Hybrid Control Architecture (MACHA) [9, 10, 11, 1] provides an effective procedure for near optimizations of difficult resource decisions.

Static Analysis.

Static "actual cost analysis" is widely used in industry [19] for planning and budgeting. It is based on historical experience tables. All historically based analysis methods have the defect that there are never any strictly similar past processes with the same environment and requirements and therefore their application has to be based on untested, and usually unstated, judgement calls. A priori values assigned at the beginning are usually not updated as the processes responsible for budget decisions evolve. Moreover the models often use assumed forms of statistical distributions of variables which have not been measured. The defects in the planning process are detected retrospectively by tracing back failures in the results to their causes in poor decisions.

No Feedback.

There is a general lack of feedback or self-correcting mechanisms. This is due to a lack of even the most elemental dynamic modelling of the complex interactions between decision makers as they affect the gen-

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eration of economic actions. Unlike dynamic models, actual cost analysis cannot take into account nonlinearities due to feedback between agents and processes, or even between economic variables. Finally, near-optimal plans and budgets for components cannot be constructed based on static analysis of the purely local information available to individual decision makers because of the mutual feedback effects of their ongoing decisions. In summary, in static analysis the dynamics of the evolution of the processes being budgeted for are not modelled and are not employed.

Necessity of Global Dynamic Analysis.

If we wish to achieve global goals for systems of partially autonomous interacting distributed economic decision makers, we have to model their interactions with each other and the environment, their global goals, and the dynamics of the evolution of the processes. As outlined above, this is beyond the capabilities of current actual cost analysis schemes.

Can we develop a common framework which will allow us to compute robust and near optimal plans and decentralized budgets to meet global organizational goals? These issues are partly addressed by the application of theoretical and applied economics to planning and decentralization. Mathematical models and experience tables for planning and optimization by a given economic agent in a *homogeneous* model or game are the standard stuff of operations research, management science, and applied economics. But how do we optimize and plan for economic organizations comprised of heterogeneous economic agents? There has been, up to now, no cost/benefit software which incorporates heterogeneous organizational subunits. These are subunits will have different optimality criteria for different agents and different behavioral cost functions for different agents. Even for large homogeneous organizations, current algorithms and models cannot be executed fast enough to update distributed budgets and plans on-line. The Kohn-Nerode algorithms, based on the relaxed calculus of variations and differential geometry, are designed to overcome these computational obstacles.

We propose a new architecture to remedy the problems with static actual cost analysis which we have outlined above. Our new architecture is based on hybrid systems theory and dynamic games. In general, a game is characterized by a collection of rules. When human agents are involved, interaction always involves some level of competition. The formulation of this interaction ranges from formal games to situations where rules are difficult to discern. Social scientists and economists both have endeavored to characterize significant cases of competition by abstractions which select rules and hence games. Many such games have been carefully studied mathematically, defining possible strategies and giving deep insight about many forms of human competition [3]. We have taken the Von Neumann and Morgenstern point of view that, in the large, human agents rarely apply "pure" strategies, but rather compete by combinations of strategies fashioned from their own unique points of view and understanding of the course of play. We have meshed this with MAHCA [6, 1], whose control policies (plans and strategies) are indeed "chattering" (mixed) combinations of pure strategies.

We believe that in [1] we were the first to propose a single implementable and principled mathematical model for integrating interacting economic agents which takes into account their rules of negotiation, differing cost functions and behavioral models, and the global organizational goals. Such a model can be used to compute, in real time, unit plans and budgets which are cost effective relative to the assumed behavior of the units, without applying any centralized control, and assuring that global goals and constraints are met.

The principle benifit of using the Multiple Agent Hybrid Control Architecture (MAHCA) is that it allows us to use a cost/benefit model in which the subunits of a heterogeneous economic organization and their interactions constitute a partially decentralized "Cost/Benefit" economic game played between subunits. The MAHCA model then becomes a dynamic economic game played between players with heterogeneous cost functions and with both logic and continuous evolution constraints. The budgets so obtained are"decentralized" plans for individual subcomponents that meet material, financial, informational, and behavioral constraints of the heterogeneous organization. MAHCA algorithms assure that for a given the organizational structure, the plans and budgets computed are robust and cost-effective or, using a more technical term, near optimal.

Economic models with rules for interaction and heterogeneous competitive strategies have been studied for many years in economics for manufacturing, legislative, executive, enterprise, and other processes. The Multiple Agent Hybrid Control Architecture for the first time allows the extraction of heterogeneous dynamic game strategies simultaneously for all the agents at once. These games are played between multiple agents which participate in economic policy modeling and game law extraction. The composite strategy coming from the application of the individual strategies, generates a near minimum cost solution for the implementation of design initiatives.

Moreover this behavior is obtained in the presence of knowledge uncertainties, such as incomplete models of the systems being designed, loosely defined operational requirements, and evolving design parameters.

2. Problem formulation

In this section, we formulate the basic game problem that we shall solve. Suppose that there are N players (herein referred to as agents) ($N > 1$). As a function of interdependency, which is determined by the application, we group the N players into not necessarily disjoint subsets $S_1, \ldots, S_k$. Within each subset, the players play a common game according to a set of rules prespecified for the subset. Notice that since the subsets S_i are not necessarily disjoint, a given agent may be playing in more than one game. This arrangement implies that the action of an agent has to *simultaneously* comply with game rules corresponding to more than one game. In conventional formulations, this leads to inconsistencies. However, if we are willing to allow as a solution of our games a *two-time scale chattering solution*, we obtain implementable game strategies for this problem. In the next paragraphs we will overview the nature of this solution which is the basis of our proposed cost analysis implementation.

An example of a game formulated cost analysis system of the type introduced above is shown in figure 1. The system has three agents controlling the (cost) process. Each agent observes the process via its localized sensors and acts on it via its localized actions. In addition, the agents interact dynamically with each other via an interagent network. The three agents are playing two simultaneous games: S_1 involving agents 1, 2, and 3 and S_2 involving agents 2 and 3. Within each subset, the agents are suppose to conduct a game whose effects are the generation of actions that modify the behavior of the process. The nature of these actions will be discussed later in this section. For now we proceed to overview the main characteristic of our proposed solution.

In a two-time scale chattering solution, each agent contends with two hierarchically organized time scales: the game scale and the action scale. This is illustrated in figure 2. Each game step takes Δ seconds. Each agent must determine what portion of this interval will be assigned to play according to one of a finite set of basis games. Then, within each subinterval the agent has to decide the time it assigns to one of a finite set of pre-established primitive infinitesimal actions [7].

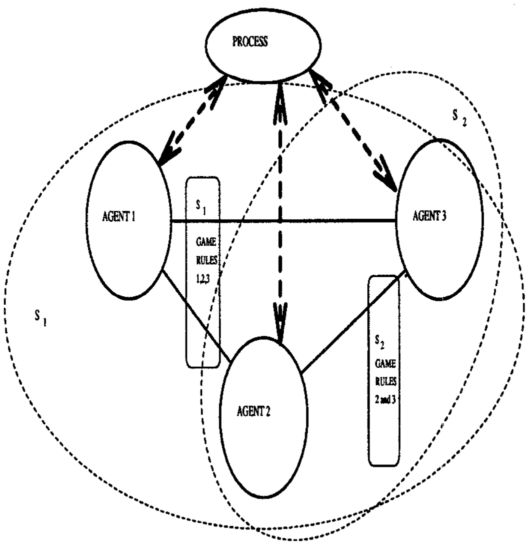

Figure 1: Example of a Multiple Game Arrangement.

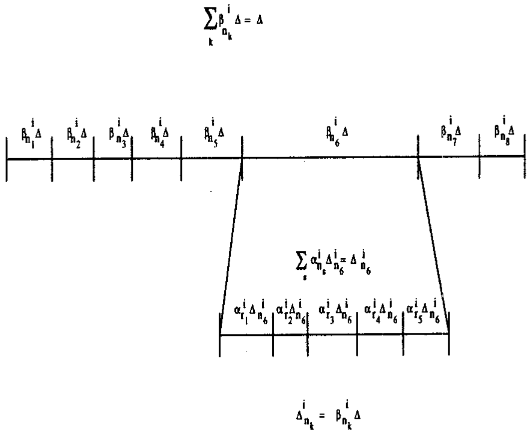

Figure 2: Two-time Chattering for Agent i.

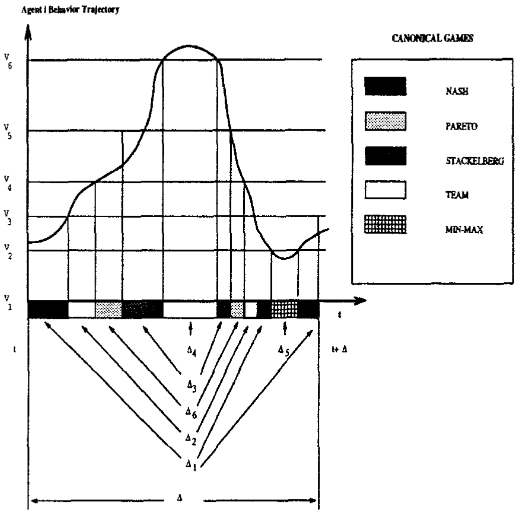

Figure 3: Basis Game Chattering.

Each basis game is characterized by a different (rule) model. The heterogeneous game is carried out by implementing each game agent with its associated game and process rules as a MAHCA agent. As illustrated in figure 2, a MAHCA agent is performing on-line two hierarchically organized functions: (1) the construction of a composite game strategy by game chattering combination and (2) the generation of infinitesimal actions that are chattering combinations of the primitive infinitesimal actions available to the agent. The other principle feature of MAHCA, adaptation, works similarly as the adaptation process that is used in control problems [7]. Because of space limitations, adaptation will not be discussed in this paper, except to say that deviations from expected conservation laws are used to modify the cost functions that the agents optimize.

The basic idea behind our proposed solution of any particular game problem in which an agent is involved is to construct an arbitrarily close behavior to the optimal behavior of the game by chattering among behaviors of a suitably selected basis for the game. The solution of each basis game is obtained by chattering of game actions as in control problems [4]. This is illustrated in figure 3.

3. Conclusion

The central results of this paper is that a composite strategy coordinating the agents for achieving a near optimal cost solution can be obtained by an extension of the Kohn-Nerode procedure for near optimal control law extraction [10, 11, 12] to the near optimal solution of the dynamic cost analysis problem. Our methodology [1] has as intermediate step the construction of game rules that satisfy all multiple interagent constraints and the construction of behavioral and cost models for the individual agents. In these models, the concept of a "carrier manifold" and its connections plays a basic role, see [12]. In this paper we formulated cost analysis as a system of agents which play differential, rather than discrete, games. By our continualization procedure [12], any discrete game can be transformed into an ϵ-approximate differential game, so this is no limitation. A complete exposition of these methods will be published in full elsewhere.

Finally, we are in the process of constructing game models for solving a variety of real time industrial, business, and military planning and budgeting problems.

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