

# Reactive Control of Distributed Interactive Simulations

**Wolf Kohn\*** and **Jeffrey B. Rummel†**

HyBrithms Corp.‡

1201 SE 8th Street #J140

Bellevue, WA 98004-6420

**Anil Nerode§**

HyBrithms Corporation and

Mathematical Sciences Institute, Cornell University

Ithaca, NY 14853

## 1 Introduction

A network of distributed interactive simulations is a hybrid system, that is, a system described by an amalgamation of logical and evolution models. The type of applications that motivated our work here is a distributive interactive simulation initiative of the US Army to create a realistic warfare training environment by using high speed communications to permit real-time interaction between geographically dispersed warfare simulations, with a wide range of capabilities and scale of operation, with actual combat units in the field, see [56, 57, 58]. Other applications include cooperative game playing over the Internet, virtual manufacturing and virtual enterprises. We will refer to all these types of applications by the generic name of Distributed Interactive Simulations (DIS).

The importance of realistic training was emphasized by General George C. Marshall to Congress before World War II [58]. Congress supported a sequence of large-scale maneuvers then, starting with the Louisiana Maneuvers in the spring of 1940. More recently, the Army created a new Louisiana Maneuvers (LAM) initiative to make decisions about future doctrine, force mix, force composition, and other fundamental issues. The last twenty years' efforts resulted in

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\*Research supported by Dept. of Commerce Agreement 70-NANB5H1164. e-mail: wk@hybrithms.com

†Research supported by Dept. of Commerce Agreement 70-NANB5H1164. On leave from the Department of Mathematics, University of California at San Diego. e-mail: jrummel@ucsd.edu

‡Formerly known as Sagent Corporation

§Research supported by Dept. of Commerce Agreement 70-NANB5H1164. e-mail: anil@math.cornell.edu

many successful simulations for various levels of command and different granularities of precision in modeling battlefield dynamics. Simulation of multiple levels of battlefield dynamics is accomplished by the Corps Battle Simulation (CBS) system for training at the corps/division level and by the Brigade/Battalion Battle Simulation (BBS) system for the brigade/battalion level. However, the National Simulation Center evaluation of their software designs and hardware architectures is that they limit realism. The Warfighter's Simulation 2000 (WARSIM 2000) was established to improve the effectiveness of training by dramatically increasing the realism and scope of the available training environment. The intent is to evolve a system which will synchronize and resolve the interactions of many different entities: e.g., a unit, a weapons system, a minefield, an operation's phase line, or other objects. WARSIM 2000 will use a DIS Network to interface with other simulated and actual units such as the Standard Theater Army Command and Control System (STACCS), Battle Command Training Program (BCTP), Combined Arms Tactical Trainer (CATT), and Battlefield Distributed Simulation-Developmental (BDS-D). WARSIM 2000 is anticipated to be operational in the late 1990s.

In this paper we will discuss a proposed distributed control architecture for a DIS architecture which allows for the dynamic integration and communication of interacting processes using intelligent control agents cooperating via a logic communication network. This architecture, termed the Multiple Agent Hybrid Control Architecture (MAHCA) is based on our recent advancements in hybrid systems theory and applications. This approach provides for the flexible interoperability of distributed, real-time information systems by generating, in real time, control programs which comply with logical and evolutionary specifications.

## 1.1 Technical barriers to DIS

The type of Distributed Interactive Simulation network we envision is a large-scale, dynamically scalable aggregation of sets of geographically-distributed, highly diverse system simulations interacting over a wide-area, high-speed communication channel. There are a number of the technical issues that must be addressed before such a system can be implemented at an acceptable cost. For example, for the Army DIS system, the system must be dynamically scalable. That is, we must allow individual simulations to be connected and disconnected during a single virtual session. The system must maintain a consistent, accurate view of the synthetic process between simulations with dissimilar communications protocols, data representations, scope of actions, decision time scales and levels of abstraction. The system must provide a framework for the integration of new simulation hardware and software including system verification. Of course such issues will also be important, to a greater or lesser degree, in other DIS applications as well. Among some of the issues which will come up in all DIS are realism, synchronization, verification, and scalability.

**Realism:** In a DIS environment, one strives to achieve realism relative to every user, meaning that a user's expected perception of reality and what is

delivered to the user by the network are in agreement module some accepted relaxation. We do not propose to deliver global realism because except for trivial scenarios that is not possible. For example, in the Army DIS system, the goals are the evaluation of operational alternatives and training commanders, staffs, and weapons crews on the concept of operations and the operational environment. Synchronization of operations in training and in war depends on a common view of the local battlefield environment appropriate to the level of command (i.e., consistent local views from weapons systems level through higher-level command). One of the known technical shortfalls of the current Army DIS architecture in creating and sharing such a set of views is the inability to correlate environments (e.g. terrain, vegetation, ocean bottom, weather, clouds, communications) at multiple levels of aggregation (so that a vehicle being “driven” from Fort Knox goes behind a hill at the National Training Center but does not lose line-of-sight communications, or a vehicle generated by a Semi-Automated Forces program drives smoothly over a 5-foot-deep, 30-foot-wide crater “created” by a cratering charge placed by another program) [54]. This lack of correlation between elements in the battle theater decreases realism in representing the effects of terrain and communications on military operations. The current approach to a solution of the problem is to identify critical parameters, coordinate agreement on a distributed communications mechanism, and implement simulations in a common synthetic environment. This approach supports the creation of experiments and the conduct of tests only for a single solution based on current technology to integrate constructive and virtual simulation with live simulations. However, this traditional engineering approach to integration of logical and evolution models will not support cost-effective solutions to integration of diverse methods for achieving spatial and temporal consistency for Advanced Distributed Simulations (ADS) as the technology evolves over time.

**Synchronization:** A basic need for expansion of the training capabilities present in the existing DIS architecture is the ability to play “what-if drills” with coalition operation alternatives, including peacemaking, peacekeeping, and a range of operations other than war. The synchronization of operations that will make this possible will require substantial horizontal technology integration. Synchronization of operations depends on what is now, according to General Gordon R. Sullivan, the most difficult thing to do: “get a common picture, a common view of the battle” [58].

**Verification:** One of the most challenging problems facing the successful implementation of DIS systems is the verification of software systems that use both logical (e.g. human behavior representation) and evolution (e.g. environmental representation) models. These models use fundamentally different mathematical tools. Cognitive models are built using linguistic (in the Computer Science sense) tools which depend on models built using the tools of discrete mathematics, e.g. grammars, difference equations, logic, etc. Models of the physical environment are built using simulation tools which support experiments with compositions of both discrete mathematical based, linguistic (logical) models and continuous models involving differential equations, normed

vector spaces, etc. This dichotomy is characteristic of hybrid systems. Determining the behavior of the composition of models for safety, reliability and performance constraints requires experimentation. The general bugs in the software until reaching a comfort level and declaring success. This leaves a nagging expectation that not all of the states of the computer finite-state machine have been visited and tested. And, when new capabilities are added, new failure modes are created so the whole testing process must begin anew. The designs of the existing CBS and BBS simulations makes software modification and validation of software interoperability a lengthy, resource-intensive, medium-risk process. Current verification strategies work well only where the systems are small and extensive testing is feasible. For systems the size and complexity of the Army DIS, exhaustive testing to verify performance for every state of the system finite-state machine is not possible; it is simply not feasible to execute every path under every possible failure. In addition, every modification creates the possibility of new failure modes and thus requires new testing.

**Scalability:** A DIS architecture must provide a means of explicitly representing multiple time scales of action and for rapid reconfiguration of the representation of environment, resources, and strategies to enable experimentation with alternative scenarios. To provide the necessary levels of realism, the DIS must further provide a consistent visualization across multiple levels of abstraction, and provide reliable, timely interaction between existing and future simulations. Current ‘state-of-the-art’ design does not provide for the ready integration of simulations which must comply simultaneously with both logical and evolution constraints.

## 1.2 A multiple-agent distributed architecture for the DIS

To meet the demands of future DIS applications, we assert that a fundamental improvement in control of the distributed interactive processes is required. In the following sections we provide a multiple-agent distributed control architecture for the dynamic integration and communication of DIS processes through a communication network. This architecture is based on the mathematical foundations of hybrid systems theory, developed by the authors [21, 9, 26, 29, 32, 34, 35, 39, 40].

It is not possible to apply hybrid systems theory to existing simulations since rewriting these systems would be an enormous task. Instead, we propose to apply the hybrid systems theory to the aggregation of existing simulations within an architectural framework supporting the dynamic reconfiguration of the system, in response to changes of the synthetic process, by the addition or deletion of real or simulation agents and the reallocation of communication resources.

Our approach is to customize our existing reactive, adaptive, real-time control architecture, termed Multiple-Agent Hybrid Control Architecture (MAHCA) [26, 33, 27] to control the interaction between heterogeneous, distributed simulations. The customization consists chiefly of assigning one or more decision makers, or “agents,” to each simulation involved in a DIS process, see Figure 1,

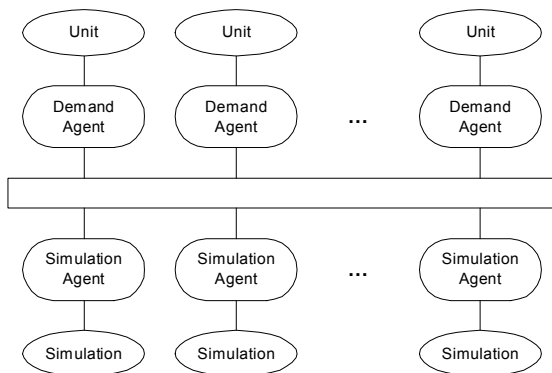


Figure 1: Multiple-Agent Hybrid Control Architecture for DIS

and to load each agent’s knowledge base with logical and equational information about the simulation assigned to it and about primitives for interaction with other agents. Each agent maintains knowledge of both the state of the simulation, or unit, and of the synthetic process at an appropriate level of abstraction. The agent responds to requests from the simulation, or unit, and provides information to other agents on the net as necessary.

MAHCA implements a dynamic, distributed optimization procedure in which each agent has a cost function and both logic and continuous evolution constraints. Optimum strategies, i.e., feasible plans for all active simulations and users, and inferred by the MAHCA algorithms. Plans and actions are determined simultaneously. The plans so obtained are “decentralized.” The MAHCA algorithms also infer required rates of communication for the links in the logic network of agents. MAHCA algorithms are assured to produce plans that are robust and near optimal.

MAHCA is a natural and effective blend of logic programming, the relaxed calculus of variations, infinitesimal differential geometry, and automata theory. Logic Programming is used to express doctrine as a set of facts and rules, and as an inference engine to produce recommended control actions for subunits. The calculus of variations is used for inter-temporal optimization of continuous quantities. The relaxed calculus of variations is the basis for optimization of processes whose evolution has mixed continuous and discrete logical elements, such as costs of deployment and fixed doctrine, since in it one can express constraints of both kinds in identical terms. Infinitesimal differential geometry is the language appropriate for expression of the manifolds on which optimizations take place, for calculation of control actions in the Lie algebra of derivations, and generally for symbolic calculation of optimal plans and budgets. Automata theory is incorporated because near-optimal plans turn out to be calculable using the automata equation calculus of Eilenberg [6].

The underlying mechanism in MAHCA is the computation, on-line, of sym-

bolic solutions for a wide variety of non-convex distributed cost minimization problems. The architecture is a network of distributed agents (subunits); each agent having a local cost function. The agents communicate to other agents only corrections for the agent cost function. Each agent acts at all times to optimize its own cost function using the MAHCA algorithms at its local computer.

Contrary to the older variational calculus and operations research techniques, relaxed plans and control action strategies that are optimal almost always exist on the carrier manifold. But there is a real ‘catch.’ That is, such optimal plans recommend, based on your current state, not as a specific control action (these are recommended changes in rates of the variables of the simulation), but a probability distribution of control actions instead. But an agent in a state has to take a specific control action, that is, make a non-fuzzy decision. What can this recommended probability distribution on control actions possibly mean? For example, if two control actions “c” and “d” are recommended for the next interval  $T$  of time with respective probabilities  $1/5$ ,  $4/5$ , and a decision as to control action has to be made, what do you do? The inability to answer how to interpret and implement such recommendations as this held up the implementation in applications of the relaxed variational optimal solutions for at least twenty years. The answer was discovered by Kohn, and subsequently developed by the authors, and is the basis for MAHCA.

What the MAHCA algorithms reveal is that one should relinquish the search for the ‘Holy Grail’ of implementing a perfectly optimal plan achieving minimal cost, which is what the recommended probability distributions represents. Instead, we implement an approximation to that plan. That is, we lower our expectations and specify what deviation from minimal cost is acceptable for each agent. Then the theorems and symbolic algorithms supporting the MAHCA kernel will compute plans for each agent that make a definite decision (control action) as to which control action to take for the next time period  $T$  based on what state you are in. If that suboptimal plan is followed, a trajectory evolves for each agent within the desired tolerance of minimality for that agent’s cost function, and which meets global goals and constraints. But there is a subtlety, namely, a change of scale is needed, concerning is an atomic control action. For example, if, according to the optimal plan, the recommendation is the probability distribution on actions described before, then the definite decision action of the nearly optimal plan is to recommend a time interval  $T$  over which to perform the next control action and to choose the control action over that interval as follows. Divide the interval  $T$  in the ratio one to four into two subintervals. Use the control action assigned probability  $1/5$  for the  $1/5 T$  length subinterval, control action assigned probability  $4/5$  for the  $4/5 T$  length subinterval. (If  $T$  is sufficiently small, order does not matter.) This is called a “chattering control” in the control literature. The algorithms start with the allowable deviation from optimality and eventually compute the required  $T$  and the finite distribution of control actions as above, which guarantee the deviation is not exceeded. The change in scale is that the recommended action for the next interval  $T$  is a sequence of short control actions, not a single control action of duration  $T$ . This use of refined scales of control is the source of strength of

the relaxed variational calculus and of MAHCA.  $T$  and the sequence of short controls used over  $T$  computational construct rests on the mathematical relation Kohn-Nerode have established between probability-valued controls and chattering controls [8, 9]. It is based on convex analysis and a generalized form of dynamic programming. This is the central mechanism behind the implementation of real-time computing in MAHCA. The feature of MAHCA which permits us to implement real-time computing of plans and establish MAHCA advantage over other methods which might be used, based on hybrid dynamic programming [35], has a highly technical description. This is that the recent analysis of the underlying evolution manifold of the system, which we call the carrier manifold [34], shows that as one moves along an optimal trajectory on that manifold in a field of extremals, there is a linear connection that goes with the motion on the extremal field. The Christoffel symbols for this connection can be computed from the Bellman-Hamilton-Jacobi equation of the calculus of variations by comparing tangent planes at infinitely near points on the extremal to get a linear invertible transformation of tangent planes at the two infinitely near points. It is to be noted that these are not connections on the whole manifold in the usual sense. They parallel transport along extremals in an extremal field, but not along arbitrary curves. This can be thought of an extension of the Caratheodory view of variational calculus as the theory of an extremal field. Our embedding of the optimization problem allows us to apply this optimization techniques to both physical and cognitive models.

The relation obtained is an infinitesimal relation, that is, a single symbolic formula that can be computed in advance using generalized Christoffel symbols. Since we handle control actions entirely by the infinitesimal method as generators of flows or derivations, we can integrate the infinitesimal relation for the connection (a generalization of the geodesic equation of Riemannian geometry) to transport the desired control at the end process when the goal should be met all the way back to the present time to compute a current control, that is, to compute currently needed plans. It is the fact that the infinitesimal relation for the connection can be computed symbolically in advance all at once, that reduces on-line computation when goals change and plans and budgets have to be changed in response. Parallel transport does the trick, in almost completely precomputed symbolic form. We have found only a few special mentions of connections in the control literature. They all observe that there is a connection associated with the control found in some other way. These are in mechanics and are connections in the usual sense, allowing parallel transport on all curves on a manifold, not just on extremals as ours do.

MAHCA provides a knowledge-based, formal implementation framework for deducing on-line feedback control and reactive strategies for processes involving multiple decision makers (or agents) through the implementation of a constructive algorithm for building automata which simultaneously comply with logical and evolution equations. Each agent of the architecture has the capability for structural adaptation as a function of predictable and unpredictable events and, therefore, can satisfy quality of service and performance requirements in the unavoidable presence of sensory and knowledge uncertainty. Furthermore, if the

logic or evolution models are not completely compatible with the system they are modeling, the architecture provides for formal mechanisms for detecting the incompatibility and for tuning the structure of both the logic and evolution models.

The multiple-agent architecture provides a formal framework for a dynamically reconfigurable distributed information system cooperating without an umpire. Agents can be dynamically spawned or removed from the net. Each agent maintains an appropriate level of knowledge about other agents on the net. In addition, individual agents are self-aware, in that the knowledge base contains information on the hardware capabilities and current status. The capability information from each agent within the architecture is communicated to every other agent. This net wide self awareness provides the capability to adapt to system changes, whether it is the spawning of new agents, introduction of future simulation technology or failures of an existing simulation.

Our multiple-agent control architecture encapsulates each active element on the net. The agents are “trusted” in that they prevent any action by the simulation, or unit, from violating the integrity of the system. To the extent that the high-level (logic) models are “trusted” and low-level (evolution) models are “trusted,” we can construct automata which are consistent with the constraints of both models. Testing to determine system equilibrium is still required as is the need to experimentally verify that the high-level logic meets the needs of the users. However, the need for exhaustive testing to experimentally analyze the result of combining high-level logic and low-level evolution representations is not required.

The rest of this paper is organized as follows. In Section 2, we discuss our generic model of a DIS process based on a general conservation criterion. In Section 3, we present the salient features of a customized version of the MAHCA as the “intelligent control” for DIS. In section 4, we will discuss our approach to various synchronization and communications issues which arise in a DIS application. Finally in section 5, we present some preliminary conclusions.

## 2 Process Model

Our model for DIS processes bulids on the current hierarchical organization of the network operation, adding a logic distribution of decision agents for controlling the simulation processes. We shall discuss several aspects of this model in the section, namely the hierarchical organization of the model, the ability of the network of DIS agents to spawn new agents and eliminate existing agents, and the underlying mathematical model of a MAHCA agent.

### 2.1 Hierarchical organization

The hierarchical organization of the network system is similar to the ISO model, see Figure 2, and conforms to the evolving vision of the DIS architecture [54]. The bottom of the hierarchy contains the Physical Network, composed of the

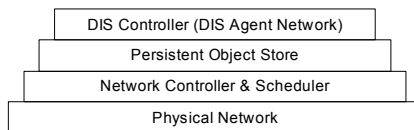


Figure 2: Hierarchical Organization of the network

primary and secondary network links, switching nodes, interfaces, etc. The next level is the Network Controller and Scheduler which processes requests from the network for DIS services and queries the data base system for setup and status information. It also receives state information from the network and processes it to generate updates to an aggregated operational network model that resides in the database.

The Persistent Object Store System (POSS) is a dynamic data base providing mechanisms for data store, data flow and presentation at a level of aggregation compatible with the needs of the DIS Controller. This database includes interactive interfaces with the Network Controller and DIS Controllers. The data base also includes entries which comply with the current standards and needs of specific simulations. More generally, data base should also include entries required to support flexible interoperability of reconfigurable simulations. For MAHCA, the objects in the database are simulation generated trajectories that are assembled from generic primitives in accordance with the DIS Controller action commands.

At the top of the hierarchy, the DIS Controller generates control actions to initiate, facilitate, monitor, and adaptively regulate DIS processes, see Figure ???. In order to accommodate the flexible reconfiguration of interoperable simulations, we propose that the network be controlled by a network of autonomous agents, the DIS Agent Network. The DIS Agent network is composed of a (varying) number of devices called Decision Agents (“agents”) implicitly coordinated through a variable (over time) configuration Logic Communication Network implemented through the hierarchy of Figure 2.

## 2.2 Dynamic Distribution of Decision Agents

The DIS controller domain of action is a hybrid distributed model of the active DIS processes throughout the network. This model is termed hybrid because its structure is an amalgamation of logic and evolution elements, see [32, 33, 34]. The evolution elements are encoded in the network model stored in the POSS relational data base. The logic elements are encoded in the knowledge base of the individual agents. We propose to model the DIS process dynamics in terms of the conservation of the flow of a commodity called Simulation Service Demand (demand for short) through the logic communication network. This network is composed of connection links and agents. The agents are allocated at the nodes of the network. As a function of time  $t$ , the network configuratio

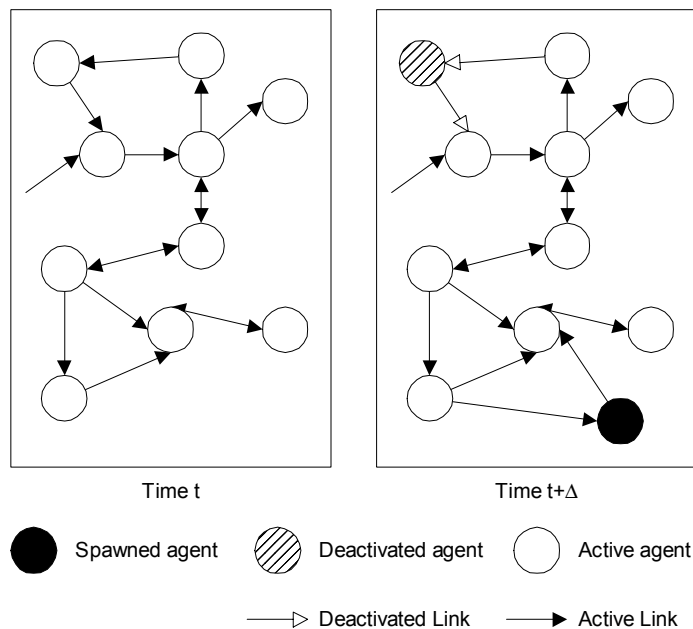


Figure 3: Logic Communication Network: Transition example

varies because new agents are spawned and become part of the network or old ones drop out.

This is illustrated in Figure 3. In Figure 3 circles represent the agents and directed edges represent the logic links between them. We emphasize that the logic connections between the agents are actually mediated through the protocol associated with the hierarchy of Figure 2.

The agents in the logic communications network are of two types: Permanent and Transitory agents. Permanent agents are those associated with DIS processes. They remain active even if the process they control becomes inactive. Transitory agents, which are spawned by the permanent agents, remain part of the network as long as the process in which they participate is active. Transitory agents are spawned as a response, among others, to increased demand, satisfaction of service requirements, and network malfunctions. In Figure 3, edges attached to only one node represent flow of demand in or out of the network.

With MAHCA algorithms, the messages passed between agents in the logic communication network are not raw data. All messages to and from an agent are mainly single corrections for a cost function that the agent or subunit uses for its local optimization. Each agent is given adequate local computing power to solve approximately its cost minimization problem in real-time based on these communicated corrections. The communications bottleneck created by massive raw data transfer is thus avoided by increasing local computer intelligence for

each agent. An agent’s computing power required to employ MAHCA is now relatively inexpensive; requiring only a single workstation for most simulations. Distribution of intelligent optimization is our method of addressing the information flow bottleneck for dynamic distributed DIS synchronization and control.

### 2.3 Model Overview

At each time  $t$ , the state of the network is given by a point in a locally smooth manifold  $M$  [7]. We refer to  $M$  as the *carrier manifold* of the DIS system. The carrier manifold is a dynamic database that captures the hybrid nature of distributed simulation processes. In general, a hybrid system has a hybrid state, the simultaneous dynamical state of all plants and digital control devices. Properly construed, the hybrid states will form a differentiable manifold which we call the carrier manifold of the system. To incorporate the digital states as certain coordinates of points of the carrier manifold, we “continualize” the digital states. That is, we view the digital states as finite, real-valued, piecewise-constant functions of continuous time and then we take smooth approximations to them. This also allows us to consider logical and differential or variational constraints on the same footing, each restricting the points allowed on the carrier manifold. In fact, all logical or discontinuous features can be continualized without practical side-effects. This is physically correct since for any semiconductor chip used in an analog device, the zeros and ones are really just abbreviations for sensor readings of the continuous state of the chip. Every constraint of the system, digital or continuous, is incorporated into the definition of what points are on the carrier manifold. Lagrange constraints are regarded as part of the definition of the manifold as well, being restrictions on what points are on the manifold.

The evolution of a distributed simulation process is characterized by two interacting models: an Evolution Model and a Logic Model, see Figure 4. The evolution model generates certain behavior trajectories in  $M$  (functions of time taking values in  $M$ ) referred to as *demand functions*. The logic model encodes the knowledge requirements for the simulation application being executed. This includes the characteristics of the simulations involved, goals of the user(s), synchronization requirements, and communication constraints.

Associated with each point  $p$  in the carrier manifold  $M$ , there is a finite dimensional vector space called the tangent space at  $p$  and termed  $TM_p$ . The actions generated by agents are elements of this vector space. The actions, that we will describe in the next subsection, are infinitesimal feedback operators which determine the behavior trajectories in  $M$ .

The central constraint on the behavior trajectories is that they satisfy the requirements encoded in the logic model. This is equivalent to the requirement that they be continuous in the finite topology of the carrier manifold, see [26, 33, 31]. This topology is determined by the encoded knowledge. We will discuss this aspect in the next subsection.

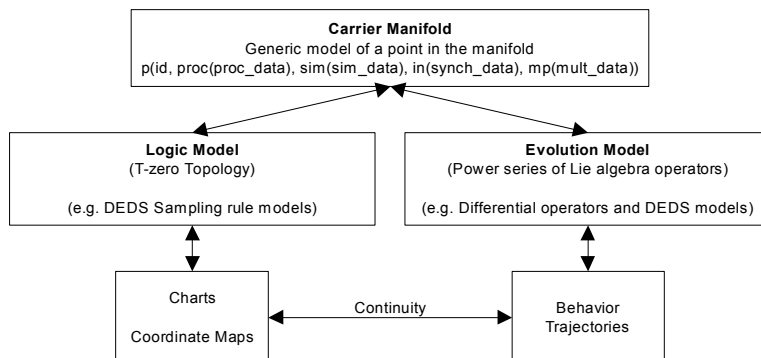


Figure 4: Carrier Manifold

## 2.4 A MAHCA Agent's Model

Let  $A_i$ ,  $i = 1, \dots, N(t)$ , denote the agents active at the current time  $t$ . In our model,  $t$  takes values on the real line. At each time  $t$ , the status of each agent in the network is given by a point in a locally differentiable manifold  $M$  by a data structure of the form

$$p(id, proc(proc\_data), media(media\_data), in(synch\_data), mp(mult\_data)) \quad (1)$$

Here  $id$  is an identifier taking values in a finite set  $ID$ ,  $proc()$  is a relation characterizing the Distributed Interactive Simulation processes status which depends on a list of parameters labeled  $proc\_data$ , which define the load and timing characteristics of the process involved. (Note that the existing simulation processes will normally have different data structures.) Next  $media$  is a relation that captures attributes of the communication processes involved in the simulation application in operation. It depends on a list of parameters labeled  $media\_data$  whose entries encode constraint instances and protocols of the application at a level of abstraction compatible with the logic communication network. Also in (1),  $in()$  is a relation carrying synchronization information of the logic communication network with respect to the hierarchical organization of Figure 2. Specifically, it characterizes the protocol at the operation point. This includes information such as priority level, connectivity and time constants. Finally, the relation  $mp()$  carries multiplicity information, that is, it represents the level of network usability at this point. The associated parameter list,  $mult\_data$ , is composed of statistical parameters reflecting the network's load.

The parameter lists in the data structure of the points of  $M$  are composed of integers, such as the number of users, reals, such as traffic loads, and discrete values, such as process identifiers or switches. They characterize the status of the network and the active processes. Computing the evolution of these parameters over time is the central task of the model.

The demand function  $D_i$  of an active agent  $A_i$  is given by a continuous

function

$$D_i : M \times T \rightarrow R^+ \quad (2)$$

where  $T$  is the real line (time space) and  $R^+$  is the positive real line.

From an agent's point of view, the dynamics of the control network are characterized by certain trajectories on the manifold  $M$ . These trajectories characterize the flow of information through the network and its status. Specifically, we need to define two items:

1. A generator for the demand functions at time  $t$ ,

$$\{D_i(p, t) \mid i = 1, \dots, I(t), p \in M\} \quad (3)$$

where  $I(t)$  is the set of active agents at time  $t$  and

2. the control actions issued by the agents.

We will see shortly that these actions are implemented as infinitesimal transformations defined in  $M$ . The general structure of the demand function in (2) for an agent  $A_i$  at time  $t$  is

$$D_i(p, t) = F_i(U_i, D, \alpha_i)(p, t) \quad (4)$$

where  $F_i$  is a smooth function,  $D$  is the vector of behavior functions,  $C_i^u$  is the unsatisfied function, and  $\alpha_i$  is the command action issued by the  $i$ th agent. We will devote the rest of this subsection to characterizing this model.

We start with a discussion of the main characteristics of the manifold  $M$ . In general a manifold  $M$  is a topological space (with topology  $\Theta$ ) composed of three items:

1. A set of points, for example in our case we will consider points of the form of (1).
2. A countable family of open subsets  $M$ ,  $\{U_i\}$ , such that

$$\bigcup_i U_i = M.$$

3. A family of smooth homeomorphisms,  $\{\phi_i \mid \phi_i : U_i \rightarrow V_i\}$ , where for each  $j$ ,  $V_j$  is an open set in  $R^k$ . The sets  $U_i$  are referred to in the literature as coordinate neighborhoods or charts. For each chart  $U_j$  the corresponding function  $\phi_j$  is referred to as its coordinate chart.

The coordinate chart functions satisfy the following additional condition:

Given any charts  $U_i$  and  $U_j$  such that  $U_i \cap U_j \neq \emptyset$ , the function  $\phi_i \circ \phi_j^{-1} : \phi_j(U_i \cap U_j) \rightarrow \phi_i(U_i \cap U_j)$  is smooth.

In the literature, one usually finds an additional property, which is the Hausdorff property in the definition of manifolds [32]. Since this property does not hold in our application we will not discuss it.

Now we proceed to customize the generic definition of the manifold to our application. We start with the topology  $\Theta$  associated with  $M$ . We note that the points of  $M$  have a definite structure (see (1)). The number of these parameters equals  $k$ . The knowledge about these parameters is incorporated into the model by defining a finite topology  $\Omega$  on  $R^k$  [7].

The open sets in  $\Omega$  are constructed from the clauses encoding what we know about the parameters. The topology  $\Theta$  of  $M$  is defined in terms of  $\Omega$  as follows. For each open set  $W$  in  $\Omega$  such that  $W \subseteq V_j \subseteq R^k$ , we require that the set  $\phi_j^{-1}(W)$  be in  $\Theta$ . The sets constructed in this way form a basis for  $\Theta$  so that a set  $U \subseteq M$  is open if and only if for each  $p \in U$ , there is  $j$  and an open set  $W \in \Omega$  such that  $W \subseteq V_j$  and  $p \in \phi_j^{-1}(W)$ .

To characterize the actions commanded by a MAHCA agent we need to introduce the concept of derivations on  $M$ . Let  $F_p$  be the space of real valued smooth functions  $f$  defined in a neighborhood of a point  $p$  in  $M$ . Let  $f$  and  $g$  be functions in  $F_p$ . A *derivation*  $v$  of  $F_p$  is a map

$$v : F_p \rightarrow F_p$$

that satisfies the following two properties:

$$v(f + g)(p) = (v(f) + v(g))(p) \quad (\text{Linearity}) \quad (5)$$

$$v(f \cdot g)(p) = (v(f) \cdot g + f \cdot v(g))(p) \quad (\text{Leibniz' Rule}) \quad (6)$$

Derivations define vector fields on  $M$  and a class of associated curves called integral curves [33]. Suppose that  $C$  is a smooth curve on  $M$  parameterized by  $\psi : I \rightarrow M$  with  $I$  a subinterval of  $R$ . In local coordinates,  $p = (p^1, \dots, p^k)$ ,  $C$  is given by  $k$  smooth functions  $\phi(t) = (\phi^1(t), \dots, \phi^k(t))$  whose derivative with respect to  $t$  is denoted by  $\dot{\phi}(t) = (\dot{\phi}^1(t), \dots, \dot{\phi}^k(t))$ . We introduce an equivalence relation on curves in  $M$  as the basis of the definition of tangent vectors at a point in  $M$  [19]. Two curves  $\phi_1(t)$  and  $\phi_2(t)$  passing through a point  $p$  are said to be equivalent at  $p$  (notation:  $\phi_1(t) \sim \phi_2(t)$ ), if there exists  $\tau_1, \tau_2 \in I$  such that

$$\phi_1(\tau_1) = \phi_2(\tau_2) = p \quad (7)$$

$$\dot{\phi}_1(\tau_1) = \dot{\phi}_2(\tau_2). \quad (8)$$

Clearly,  $\sim$  defines an equivalence relation on the class of curves in  $M$  passing through  $p$ . Let  $[\psi]$  be the equivalence class containing  $\psi$ . A *tangent vector* to  $[\psi]$  is a derivation  $v|_p$  such that in local coordinates  $(p^1, \dots, p^k)$ , it satisfies the condition that given any smooth function  $f : M \rightarrow R$ ,

$$v|_p(f)(p) = \sum_{j=0}^k \psi^j(t) \frac{\partial f(p)}{\partial p^j} \quad (9)$$

where  $p = \psi(t)$ . The set of tangent vectors associated with all the equivalence classes at  $p$  defines a vector space called the *tangent vector space* at  $p$ , denoted by  $TM_p$ . The set of tangent spaces associated with points in  $M$  can be “glued” together to form a manifold called the *tangent bundle* which is denoted by  $TM$ .

$$TM = \bigcup_{p \in M} TM_p$$

For our purposes, it is important to specify explicitly how this gluing is implemented. This will be explained below after we introduce the concept of a vector field and discuss its relevance in the model.

A *vector field* on  $M$  is an assignment of a derivation  $v|_p$  to each point of  $M$  which varies smoothly from point to point. That is, if  $p = (p^1, \dots, p^k)$  are local coordinates, then we can always write  $v|_p$  in the form

$$v|_p = \sum_{j=1}^k \lambda^j(p) \frac{\partial}{\partial p^j} \quad (10)$$

Then  $v$  is a vector field if the coordinate functions  $\lambda_i$  are smooth.

Comparing (9) and (10) we see that if  $\psi$  is a parameterized curve in  $M$  whose tangent vector at any point coincides with the value of  $v$  at a point  $p = \psi(t)$ , then in the local coordinates  $p = (\psi^1(t), \dots, \psi^k(t))$ , we must have

$$\dot{\psi}^j(t) = \lambda^j(p) \quad \text{for } j = 1, \dots, k. \quad (11)$$

In our application, each command issued by the MAHCA agent is implemented as a vector field in  $M$ . Each agent constructs its command field as a combination of ‘primitive’ predefined vector fields. Since the chosen topology for  $M$ ,  $\Theta$ , is not metrizable, we cannot guarantee a unique solution to (11) in the classical sense for a given initial condition. However, they have solutions in a class of continuous trajectories in  $M$  called *relaxed curves* [41]. In this class, the solutions to (11) are unique. We discuss the basic characteristics of relaxed curves as they apply to our process control formulation and implementation in Section 3. Next, we describe some of their properties as they relate to our plant model and control process. For this objective, we need to introduce the concept of flows in  $M$ .

If  $v$  is a vector field, any parameterized curve passing through a point  $p$  in  $M$  is called an *integral curve associated with  $v$* , if in local coordinates (11) holds. An integral curve associated with a field  $v$ , denoted by  $\Psi(t, p)$ , is termed the flow generated by  $v$  if it satisfies the following properties:

$$\begin{aligned} \Psi(t, \Psi(\tau, p)) &= \Psi(t + \tau, p) && \text{(semigroup property)} && (12) \\ \psi(0, p) &= p && \text{(initial condition)} \\ &\text{and} \\ \frac{d}{dt} \Psi(t, p) &= v|_{\Psi(t, p)} && \text{(flow generation)} \end{aligned}$$

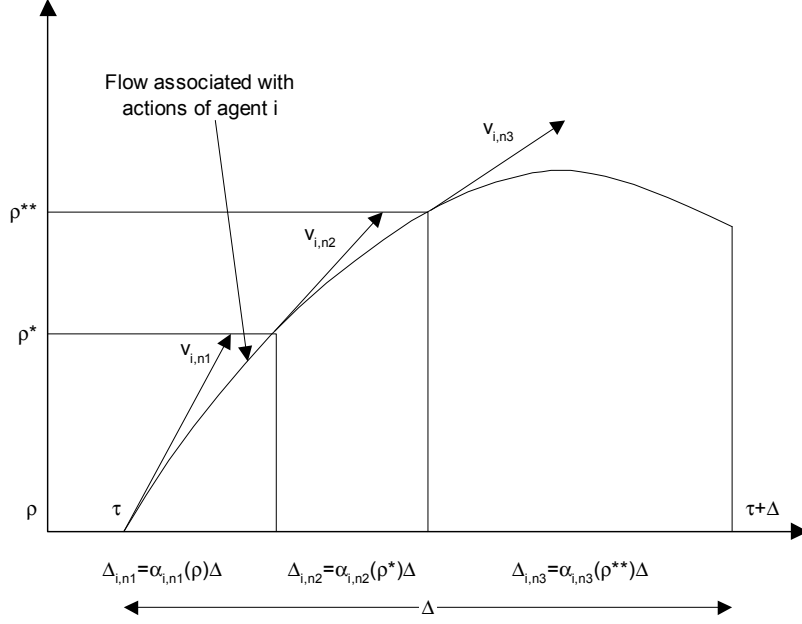


Figure 5: Conceptual illustration of agent action schedule

Now we are ready to customize these concepts for our model. Let  $\Delta > 0$  be the width of the current decision interval,  $[t, t + \Delta)$ . Let  $C_i^u(p, t)$  be the unsatisfied demand at the beginning of the interval. Agent  $A_i$  has a set of primitive actions:

$$\{v_{i,j} \mid j = 1, \dots, n_i \text{ where } v_{i,j}|_p \in TM_p \text{ for each } p \in M\} \quad (13)$$

During the interval  $[t, t + \Delta)$ , agent  $i$  schedules one or more of these actions to produce a flow which will bring the system closer to the goal. In particular,  $A_i$  determines the fraction  $\alpha_{i,j}(p, t)$  of  $\Delta$  that action  $v_{i,j}$  must be executed as a function of the external perturbation functions  $S_{r,i}(t, p)$  and the vector of the demand functions of the agents in the network  $D(p, t) = (D_1(p, t), \dots, D_N(p, t))$ . Figure 5 conceptually illustrates a schedule of actions involving three primitives. We will use this example as means for describing the derivation of our model. The general case is similar.

The flow  $\Psi_i$  associated with the schedule of Figure 5 can be computed from the flows associated with each of the actions:

$$\Psi_i(\tau, p) = \begin{cases} \Psi_{v_{i,n_1}}(\tau, p) & \text{if } t \leq \tau \leq t + \Delta_{i,n_1} \\ \Psi_{v_{i,n_2}}(\tau, \Psi_{v_{i,n_1}}(\tau, p)) & \text{if } t + \Delta_{i,n_1} \leq \tau \leq t + \Delta_{i,n_1} + \Delta_{i,n_2} \\ \Psi_{v_{i,n_3}}(\tau, \Psi_{v_{i,n_2}}(\tau, \Psi_{v_{i,n_1}}(\tau, p))) & \text{if } t + \Delta_{i,n_1} + \Delta_{i,n_2} \leq \tau \leq t + \Delta_{i,n_1} + \Delta_{i,n_2} + \Delta_{i,n_3} \end{cases} \quad (14)$$

where  $\Delta = \Delta_{i,n_1} + \Delta_{i,n_2} + \Delta_{i,n_3}$  and  $\alpha_{i,n_1} + \alpha_{i,n_2} + \alpha_{i,n_3} = 1$ . We note that the flow  $\Psi_i$  given by (14) characterizes the evolution of the process as viewed by agent  $A_i$ . The vector field  $v_i|_p$  associated with the flow  $\Psi_i$  is obtained by differentiation and the third identity in (12). This vector field applied at  $p$  is proportional to

$$[v_{i,n_1}, [v_{i,n_2}, v_{i,n_3}]] \quad (15)$$

where  $[\cdot, \cdot]$  is the Lie bracket due to the parallelogram law, see [51]. The Lie bracket is defined as follows. Let  $v$  and  $w$  be derivations on  $M$  and let  $f : M \rightarrow R$  be any real valued smooth function. The Lie bracket of  $v, w$  is the derivation defined by

$$[v, w](f) = v(w(f)) - w(v(f))$$

see [14].

Thus the composite action  $v_i|_p$  generated by the  $i$ th agent to control the process is a composition of the form of (15). Moreover from a version of the Chattering lemma and duality [24], we can show that this action can be expressed as a linear combination of the primitive actions available to the agent as follows.

$$[v_{i,n_1}, [v_{i,n_2}, v_{i,n_3}]] = \sum_j \gamma_j^i(\alpha) v_{i,j} \quad (16)$$

$$\sum_j \gamma_j^i(\alpha) = 1$$

with the coefficients  $\gamma_j^i$  determined by the fraction of time that each primitive action  $v_{i,j}$  is used by agent  $i$ .

The effect of the field defined by the composite action  $v_i|_p$  on any smooth function (equivalent function class) is computed by expanding the right hand side of (14) in a Lie-Taylor series [53]. In particular, we can express the change in the demand modification functions  $C_i^u$  due to the flow over the interval  $\Delta$  in terms of  $v_i|_p$ . The evolution of the modified demand function  $C_i^u$  over the interval starting at point  $p$  is given by

$$C_i^u(t + \Delta, p'') = C_i^u(t, \Psi_i(t + \Delta, p)) \quad (17)$$

Expanding the right handside of (17) in a Lie-Taylor series around  $(t, p)$ , we obtain

$$C_i^u(t + \Delta, p'') = \sum_j \frac{(v_i|_p(C_i^u(p, t)))^j \Delta^j}{j!}$$

where

$$(v_i|_p(\cdot))^j = v_i|_p((v_i|_p(\cdot))^{j-1}(\cdot)) \quad (18)$$

and

$$(v_i|_p)^0(\cdot) = \text{identity operator}$$

In general, the series in the right handside of (18) will have countable many non-zero terms. In our case, since the topology of  $M$  is finite due to the fact that it is generated by finitely many logic clauses, this series will have only finitely many non-zero terms. Intuitively, this is so because in computing powers of derivations (i.e., limits of differences), we need only to distinguish among different neighboring points. In our formulation of the topology of  $M$ , this can only be imposed by the information in the clauses of the agent's Knowledge Base. Since each agent's knowledge base has only finitely many clauses, there is a term in the expansion of the series in which the power of the derivation goes to zero. This is important because it allows the series in the right-hand side to be effectively generated by a locally finite automaton. We will expand on the construction of this automaton in the next section when we discuss the inference procedure carried out by each agent.

We note that given the set of primitive actions available to each agent, the composite action is determined by the vector of fraction functions  $\alpha_j$ . We will see in the next section that this vector is inferred by each agent from the proof of existence of solutions of an optimization problem.

Now we can write the specific nature of the model formulated in expression (4). At time  $t$  and at point  $p \in M$  the demand modification function of agent  $i$  is given by

$$C_i^u(p, t) = C_i^u(p, t^-) + S_{r,i}(p, t) + \sum_k Q_{i,k}(p, t) \cdot D_k(p, t^-) \quad (19)$$

where  $t^-$  is the end point of the previous update interval,  $S_{r,i}$  is the estimation request function to agent  $i$ , and  $Q_{i,k}$  is a multiplier determining how much of the current demand modification requirements of agent  $A_k$  is allocated to agent  $A_i$ . This allocation is determined from the characteristics of the process both agents are controlling and from the process description encoded in the agent's knowledge base. The actual request for service from agent  $k$  to agent  $i$  is thus the term  $Q_{i,k} \cdot D_k(p, t^-)$ . The information sent to agent  $i$  by agent  $k$  is the demand modification function  $D_k(p, t^-)$  at the end of the previous interval. Finally the point  $p \in M$  carries the current status of the process monitored by the agents appearing in (19). Agent  $k$  thus contributes to Agent  $i$ 's new control only if  $Q_{i,k} \neq 0$ .

This concludes our description of the model. For space considerations, some details have been left out. In particular those related to the strategy for activation and deactivation of agents. These will be discussed in a future paper. Now, we proceed to describe the architecture that the agents use to control the model presented in this section.

### 3 Process Control Architecture

In this section, we describe the main operational and functional characteristics of the DIS controller. As we mentioned in the introduction, this controller is implemented as a distributed system composed of agents and a communication



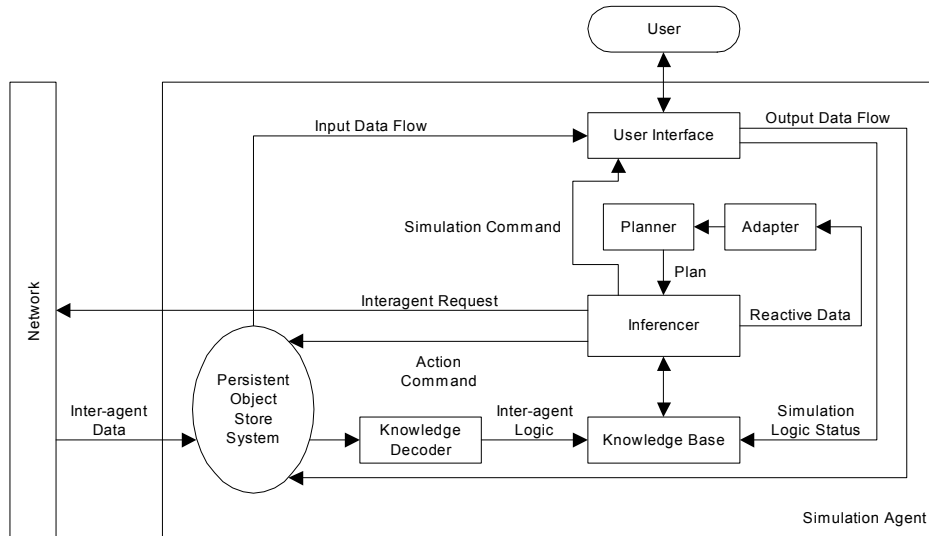


Figure 7: Simulation Agent

by the agent for a simulation or a user. The plan encodes the desired behavior corresponding to the user scenario goal for a demand agent or the desired simulation behavior for a simulation agent.

The Knowledge Base stores the requirements of operations or processes controlled by the agent. It also encodes system constraints, inter-agent protocols and constraints, sensory data, operational and logic principles and a set of primitive inference operations defined in the domain of equational terms.

The Inferencer determines whether the plan is realizable given the current status of the Knowledge Base. If the plan logically follows from the current status of the Knowledge Base, the Inferencer generates, as a side effect of proving the realizability of the plan, the current command actions. If the behavior statement does not logically follow from the current status of the Knowledge Base, that is, the plan is not realizable, the inferencer transmits the failed terms to the Adapter module for replacement or modification.

A Persistent Object Store System is the means of interfacing between an agent and the network controller and scheduler to implement the network of cooperating agents (see Figures 6 and 7). This system has a basis of generic simulation trajectory primitives that are instantiated during run time to values determined by simulation data and stitched together to form a trajectory corresponding to the behavior of the simulations. The chief advantage of having this system is that it reduces the amount of state data that has to be transmitted between agents to construct the simulation trajectories.

Finally, the Knowledge Decoder translates knowledge data from the other agents, stored in the Persistent Object Store System, and incorporates them

into the Knowledge Base of the agent.

In each agent of MAHCA, the plan is the formulation of a relaxed variational optimization problem whose successful resolution produces an action schedule of the form of (15). Each agent operates as a real-time solver of an appropriate relaxed variational optimization problem [20]. A customized version of this theory, enriched with elements of differential geometry, equational logic and automata theory provides a general representation for the dynamics, constraints, requirements and logic of the logic communications network. We devote the rest of this section to the discussion of the main elements of this theory in the context of the operational features of MAHCA.

### 3.1 Core Architectural Elements of a MAHCA Agent

We will discuss next the functionality of the six core modules of a MAHCA agent in a DIS application.

#### 3.1.1 Knowledge Base

The Knowledge Base consists of a set of equational first order logic clauses with second order extensions. The syntax of clauses is similar to the ones in the Prolog language. Each clause is of the form

$$Head \leftarrow Body \tag{20}$$

where *Head* is a functional form,  $p(x_1, \dots, x_n)$ , taking values in the binary set  $[true, false]$  with  $x_1, x_2, \dots, x_n$  variables or parameters in the domain  $M$  of the MAHCA network. The symbol  $\leftarrow$  stands for logical implication. The variables appearing in the clause head are assumed to be universally quantified. The *Body* of a clause is a conjunction of one or more logical terms

$$e_1 \wedge e_2 \wedge \dots \wedge e_n \tag{21}$$

where  $\wedge$  is the logical ‘and.’ Each term in (21) is a relational form. A relational form is one of the following: an equational form, an inequational form, a covering form, or a clause head. The logical value of each of these forms is either true or false. A relational form  $e_i$  is true precisely at the set of tuples of values  $S_i$  of the domain taken by the variables where the relational form is satisfied and is false for the complement of that set. Thus for  $e_i = e_i(x_1, \dots, x_n)$ ,  $S_i$  is the possibly empty subset of  $M^n$ ,

$$S_i = \{(x_1, \dots, x_n) \in M^n : e_i(x_1, \dots, x_n) = true\}$$

so that

$$e_i(x_1, \dots, x_n) = false \text{ if } (x_1, \dots, x_n) \in M^n \setminus S_i.$$

The generic structure of a relational form is given in Table 1.

In Table 1,  $w$  and  $v$  are polynomial forms with respect to a finite set of operations whose definitional and property axioms are included in the Knowledge

Form	Structure	Meaning
equational	$w(x_1, \dots, x_n) = v(x_1, \dots, x_n)$	equal
inequational	$w(x_1, \dots, x_n) \neq v(x_1, \dots, x_n)$	not equal
covering	$w(x_1, \dots, x_n) < v(x_1, \dots, x_n)$	partial order
clause head	$q(x_1, \dots, x_n)$	recursion, chaining

Table 1: Structure of the Relational Form

Base. A polynomic form  $v$  is an object of the form  $v(x_1, \dots, x_n) = \sum_{\omega \in \Omega} (v, \omega) \cdot \omega$  where  $\Omega^*$  is the free monoid generated by the variable symbols  $\{x_1, \dots, x_n\}$  under juxtaposition. The term  $(v, \omega)$  is called the coefficient of  $v$  at  $\omega$ . The coefficients of a polynomic form  $v$  take values in the domain of definition of the clauses. The domain in which the variables in a clause head take values is the manifold  $M$  described in section 2. The logical interpretation of (20) and (21) is that the *Head* is true if the conjunction of the terms of *Body* are jointly true for instances of the variables in the clause head. These clauses, which are application-dependent, encode the requirements on the closed-loop behavior of the model of the agent. In fact the closed loop behavior, which we will define later in this section in terms of a variational formulation, is characterized by continuous curves with values in  $M$ . This continuity condition is central because it is equivalent to requiring the system to look for actions that make the closed loop behavior satisfy the requirements of the plant model.

The denotational semantics of each clause in the knowledge base is one of the following:

1. a conservation principle,
2. an invariance principle, or
3. a constraint principle.

*Conservation principles* are one or more clauses about the balance of a particular process in the dynamics of the system or the computational resources. For instance, equation (19) encoded as a clause expresses the conservation of

demand in the logic communications network.

$$\begin{aligned}
& \text{conservation\_of\_unsatisfied\_demand}(p, t, [Q_{i,k}], S_{ri}, [D_k], \Delta, C_i^u) < \\
& \quad C_i^{u++} = \sum_j \frac{(v_i|_p(C_i^{u+}))^j \cdot \Delta^j}{j!} \wedge \\
& \quad \text{/* encoding of equation (13) */} \\
& \quad C_i^{u+} = C_i^u + S_{ri} + \sum_k Q_{i,k} \cdot D_k \wedge \\
& \quad \text{/* encoding of equation (15) */} \\
& \quad \text{process\_evolution}(p, t, p'') \wedge \text{/* encoding of equation (13) */} \\
& \quad \quad t_+ = t + \Delta \wedge \\
& \text{conservation\_of\_unsatisfied\_demand}(p'', t_+, [Q_{i,k}], S_{ri}, [D_k], \Delta, C_i^{u++}) \quad (22)
\end{aligned}$$

In (19), the first equational term relates the unsatisfied demand for agent  $i$  at the current time to the unsatisfied demand in the past and the net current demand of the other agents connected to agent  $i$ . The last term of the rule implements the recursion.

As another example, consider the following clause representing conservation of computational resources:

$$\begin{aligned}
& \text{comp}(\text{Load}, \text{ProcessOp\_count}, \text{Limit}) \\
& \quad \leftarrow \text{process}(\text{process\_count}) \\
& \quad \wedge \text{process\_count} \cdot \text{Load}_1 - \text{Op\_count} < \text{Load} \\
& \quad \quad \wedge \text{Load}_1 < \text{Limit} \\
& \quad \quad \wedge \text{comp}(\text{Load}_1, \text{Process}, \text{Op\_count}, \text{Limit})
\end{aligned}$$

where ‘Load’ corresponds to the current computational burden, measured in VIPS (Variable Instantiations Per Second), Process is a clause considered for execution, and Op\_count is the current number of terms in process.

Conservation principles always involve recursion whose scope is not necessarily a single clause, as in the example above, but with chaining throughout several clauses.

*Invariance principles* are one or more clauses establishing constants of motion in a general sense. These principles include stationarity, which plays a pivotal role in the formulation of the theorems proved by the architecture, and geodesics. In the control of DIS processes, invariant principles specify quality response requirements. That is, levels of performance as a function of traffic load that the system must satisfy.

The importance of invariance principles lies in the reference they provide for the detection of unexpected events. For example, in a DIS process, the update time after a request is serviced is constant under normal operating conditions. An equational clause that states this invariance has a ground form that is constant, and any deviation from this value represents deviation from normality.

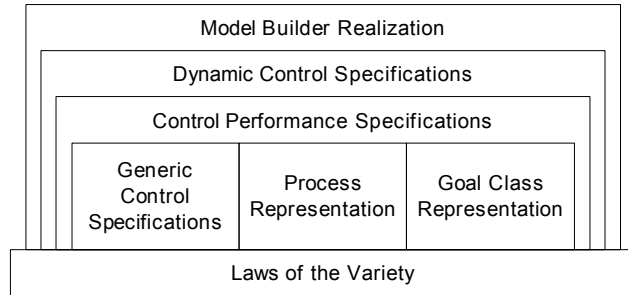


Figure 8: Knowledge Base Organization

*Constraint principles* are clauses representing engineering limits to actuators or sensors and, most importantly, behavioral processes. For instance, in a DIS process, the characteristics of the speed of response of the values controlling the input flow size or the speed of access are given by empirical graphs (e.g. byte # vs. velocity with traffic volume as a parameter) stored in the system relational base. Another example in this application is given by the clauses that define the lifting strategy for embedding discrete varying trajectories into  $M$  (interpolation rules).

The clause database is organized in a nested hierarchical structure illustrated in Figure 8. The bottom of this hierarchy contains the equations that characterize the algebraic structure defining the terms of relational forms: i.e., an algebraic variety [53].

At the next level of the hierarchy, three types of clauses are stored: Generic Control Specifications, System Representation and Goal Class Representation.

The *Generic Control Specifications* are clauses expressing general desired behavior of the system. They include statements about stability, complexity and robustness that are generic to the class of declarative rational controllers implemented by MAHCA agents. These specifications are written by constructing clauses that combine laws of the kind which use the Horn clause format described earlier.

The *Process Representation* is given by clauses characterizing the dynamic behavior and structure of the plant, which includes sensors and actuators. These clauses are written as conservation principles for the dynamic behavior and as invariance principles for the structure. As for the Generic Control Specifications, they are constructed by combining a variety of laws in the equational Horn clause format.

The *Goal Class Representation* contains clauses characterizing sets of desirable operation points in the domain (points in the manifold  $M$ ). These clauses are expressed as soft constraints; that is, constraints that can be violated for finite intervals of time. They express the ultimate purpose of the controller but not its behavior over time.

The next level of the hierarchy involves the *Control Performance Specifica-*

tions. These are typically problem-dependent criteria and constraints. They are written in equational Horn clause format. They include generic constraints such as speed and time of response, and qualitative properties of state trajectories [48].

*Dynamic Control Specifications* are equational Horn clauses whose bodies are modified as a function of the sensor and goal commands.

Finally, *Model Builder Realization* clauses constitute a recipe for building a procedural model (an automaton) for generating variable instantiation and for theorem proving.

### 3.1.2 The Planner

The function of the theorem Planner, which is domain-specific, is to generate, for each update interval, a symbolic statement of the desired behavior of the system, as viewed, say by agent  $j$ , throughout the interval. The theorem statement that it generates has the following form:

Given a set of primitive actions there exists control schedule  $v_i|_p$  of the form (16) and a fraction function differential  $d\alpha(\cdot)$  (Figure 5) in the control interval  $[t, t+\Delta)$  such that  $d\alpha(\cdot)$  minimizes the functional

$$\int_t^{t+\Delta} L_i(\Psi_i(\tau, p), v_i|_p(G_i(\tau, p))) d\alpha(p, d\tau) \quad (23)$$

subject to the following constraints:

$$\begin{aligned} g_i(S_i, \Psi_i(t + \Delta, p)) &= G_i(t, X_i) \text{ (local goal for the interval),} \\ \sum_m Q_{i,m}(p, t) L_m(p, t) &= V_i(p, t) \text{ (interagent constraint see expression (19)), and} \end{aligned} \quad (24)$$

$$\int_{[t, t+\Delta)} d\alpha(p, d\tau) = 1$$

In (23),  $L_i$  is the *Local Relaxed Lagrangian* of the system as viewed by Agent  $i$  for the current interval of control  $[t, t + \Delta)$ . This function, which maps the Cartesian product of the state and control spaces into the real line with the topology defined by the clauses in the knowledge base, captures the dynamics, constraints and requirements of the system as viewed by agent  $i$ . The relaxed Lagrangian function  $L_i$  is a continuous projection in the topology defined by the knowledge base (see [44]) in the coordinates of the  $i$ th agent of the global Lagrangian function  $L$  that characterizes the system as a whole.

In (24),  $p$  represents the state of the process under control as viewed by the agent and  $G_i$  is the parallel transport operator bringing the goal to the current interval, see [34]. The operator  $G_i$  is constructed by lifting to the manifold the composite flow (see equation (14)). We note that the composite flow and the action schedule are determined once the fraction function is known and that this

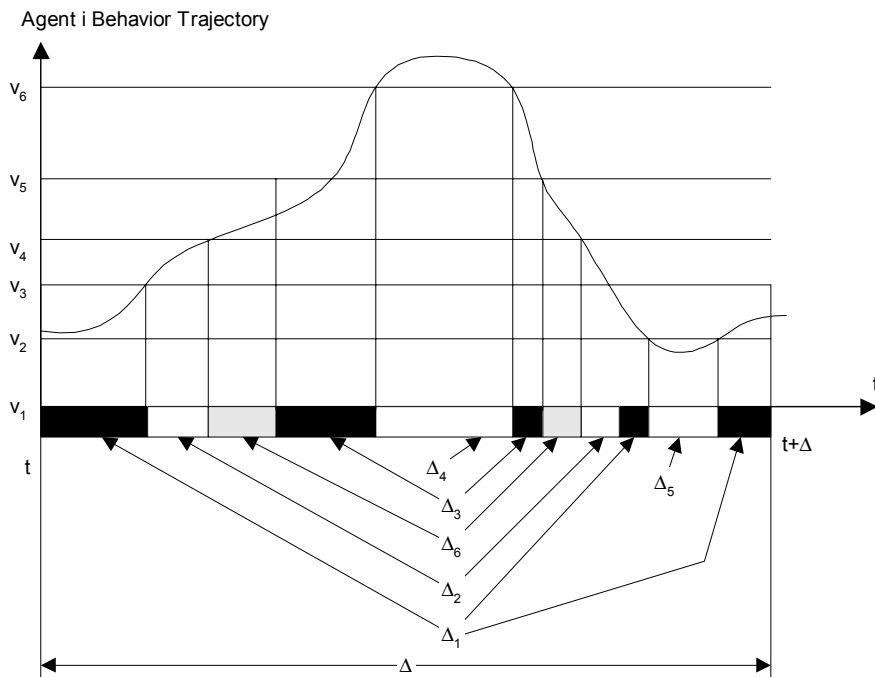


Figure 9: Illustration of optimization

function is the result of the optimization (23), (24). In particular, the action schedule is constructed as a linear combination of primitive actions (see equation (16)).

The term  $d\alpha(\cdot)$  in (23) is a Radon probability measure [53] on the set of primitive estimation actions or derivations that the agent can execute for the interval  $[t, t + \Delta)$ . It measures, for the interval, the percentage of time to be spent in each of the primitive actions. The central function of the control agent is to determine this mixture of actions for each control interval. This function is carried out by each agent by inferring from the current status of its Knowledge Base whether a solution of the optimization problem stated by the current theorem exists, and, if so, to generate corresponding actions and state updates. Figure 9 illustrates the relations between the primitive actions and the fraction of  $\Delta$  they are active in the interval  $[t, t + \Delta)$ .

The expressions in (24) constitute the constraints imposed in the relaxed optimization problem solved by the agent. The first one is the local goal constraint expressing the general value of the state at the end of the current interval. The second represents the constraints imposed on the agent by the other agents in the network. Finally, the third one indicates that this is a probability measure. Under relaxation and with the appropriate selection of the domain (see [28]), the optimization problem stated in (23) and (24) is a convex optimization problem.

This is important because it guarantees that if a solution exists, it is unique up to probability, and also, it guarantees the computational effectiveness of the inference method that the agent uses for proving the theorem.

The construction of the theorem statement given by (23) and (24) is the central task carried out in the Planner. It characterizes the desired behavior of the process as viewed by the agent in the current interval so that its requirements are satisfied and the system “moves” towards its goal in an optimal manner.

### 3.1.3 Adapter

The function under the integral in (23) includes a term, referred to as the “catch-all” potential, which is not associated with any clause in the Knowledge Base. Its function is to measure unmodeled dynamic events. This monitoring function is carried out by the Adapter which implements a generic commutator principle similar to the Lie bracket discussed in section 2. Under this principle, if the value of the catch-all potential is empty, the current theorem statement adequately models the status of the system. On the other hand, if the theorem fails, meaning that there is a mismatch between the current statement of the theorem and system status, the catch-all potential carries the equational terms of the theorem that caused the failure. These terms are negated and conjuncted together by the Inferencer according to the commutation principle (which is itself defined by equational clauses in the Knowledge Base) and stored in the Knowledge Base as an adaptation dynamic clause. The Adapter then generates a potential symbol, which is characterized by the adaptation clause and corresponding tuning constraints. This potential is added to criterion for the theorem characterizing the interval.

The new potential symbol and tuning constraints are sent to the Planner which generates a modified local Lagrangian for the agent and goal constraints. The new theorem, thus constructed, represents adapted behavior of the system. This is the essence of reactive structural adaptation in the our model.

At this point, we pause in our description to address the issue of robustness. To a large extent, the adapter mechanism of each MAHCA agent provides the system with a generic and computationally effective means to recover from failures or unpredictable events. Theorem failures are symptoms of mismatches between what the agent thinks the system looks like and what it really looks like. The adaptation clause incorporates knowledge into the agent’s Knowledge Base which represents a recovery strategy. The Inferencer, discussed next, effects this strategy as part of its normal operation.

### 3.1.4 Inferencer

The Inferencer is an on-line equational theorem prover. The class of theorems it can prove are represented by statements of the form of (20) and (21), expressed by an existentially quantified conjunction of equational terms of the form:

$$\exists Z (W_1(Z, p) \text{ rel}_i V_1(Z, p) \wedge \cdots \wedge W_n(Z, p) \text{ rel}_n V_n(Z, p)) \quad (25)$$

where  $Z$  is a tuple of variables each taking values in the domain  $D$ ,  $p$  is a list of parameters in  $D$ , and  $\{W_i, V_i\}$  are polynomial terms in the semiring polynomial algebra,

$$\tilde{D}\langle\Omega\rangle \quad (26)$$

with  $\tilde{D} = (D, \langle +, \cdot, 1, 0 \rangle)$  a semiring algebra with additive unit 0 and multiplicative unit 1. In (25),  $rel_i$ ,  $i = 1, \dots, n$  are binary relations on the polynomial algebra. Each  $rel_i$  can be either an equality relation ( $=$ ), inequality relation ( $\neq$ ), or a partial order relation. In a given theorem, more than one partial order relation may appear. In each theorem, at least one of the terms is a partial order relation that defines a complete lattice on the algebra; that corresponds to the optimization problem. This lattice has a minimum element if the optimization problem has a minimum. Given a theorem statement of the form of (25) and a knowledge base of equational clauses, the inferencer determines whether the statement logically follows from the clauses in the Knowledge Base, and if so, as a side effect of the proof, generates a non-empty subset of tuples with entries in  $M$  giving values to  $Z$ . These entries determine the agent's actions. Thus, a side effect is instantiation of the agent's decision variables. In (26),  $\Omega$  is a set of primitive unary operations,  $\{v_i\}$ , the infinitesimal primitive fields defined in section 2. Each  $v_i$  maps the semiring algebra, whose members are power series involving the composition of operators, on  $Z$  to itself,

$$v_i : \tilde{D}\langle\langle Z \rangle\rangle \rightarrow \tilde{D}\langle\langle Z \rangle\rangle \quad (27)$$

These operators are characterized by axioms in the Knowledge Base and are process dependent. In formal logic, the implemented inference principle can be stated as follows. Let  $\Sigma$  be the set of clauses in the Knowledge Base. Let  $\Rightarrow$  represent implication. Proving the theorem means to show that it logically follows from  $\Sigma$ , i.e.,

$$\Sigma \Rightarrow \textit{theorem}. \quad (28)$$

The proof is accomplished by sequences of applications of the following inference axioms:

- (i) equality axioms
- (ii) inequality axioms
- (iii) partial order axioms
- (iv) compatibility axioms
- (v) convergence axioms
- (vi) knowledge base axioms
- (vii) limit axioms

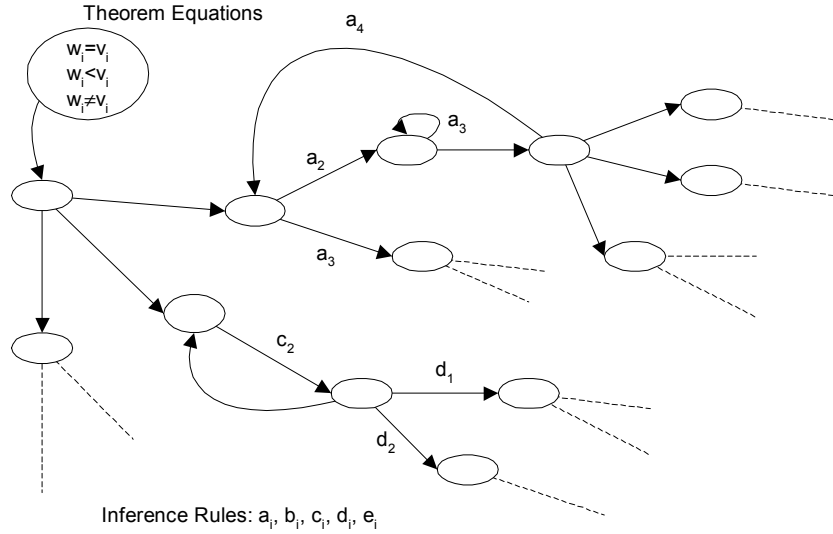


Figure 10: Conceptual Structure of the Proof Automaton

The specifics of these inference axioms can be found in [18] where it is shown that each of the inference principles can be expressed as an operator on the Cartesian product:

$$\tilde{D}\langle\langle W \rangle\rangle \times \tilde{D}\langle\langle W \rangle\rangle \quad (29)$$

Each inference operator transforms a relational term into another relational term. The inferencer applies sequences of inference operators on the equational terms of the theorem until these terms are reduced to either a set of ground equations of the form of (30) or it determines that no such ground form exists.

$$Z_i = \alpha_i \quad \alpha_i \in \tilde{D} \quad (30)$$

The mechanism by which the inferencer carries out the procedure described above is by building a procedure for variable goal instantiation, i.e., a locally finite automaton. We refer to this automaton as the Proof Automaton. This important feature is unique to our approach. The proof procedure is customized to the particular theorem statement and Knowledge Base instance it is currently handling. The structure of the proof automaton generated by the inferencer is illustrated in Figure 10.

In Figure 10, the initial state represents the equations associated with the theorem. In general, each state corresponds to a derived equational form of the theorem through the application of a chain of inference operators to the initial state that is represented by the path

$$s_0 \xrightarrow{\text{inf}_1} s_1 \xrightarrow{\text{inf}_2} \dots \xrightarrow{\text{inf}_k} s_k$$

Each edge in the automaton corresponds to one of the possible inferences. A state is terminal if its equational form is a tautology, or it corresponds to a canonical form whose solution form is stored in the Knowledge Base. In traversing the automaton state graph, values or expressions are assigned to the variables. In a terminal state, the equational terms are all ground states (see (30)). If the automaton contains at least one path starting in the initial state and ending in a terminal state, then the theorem is true with respect to the given Knowledge Base and the resulting variable instantiation is a valid one. If this is not the case, the theorem is false. The function of the complete partial order term present in the conjunction of each theorem provable by the inferencer is to provide a guide for constructing the proof automaton. This is done by transforming the equational terms of the theorem into a canonical fixed point equation, called the Kleene-Schutzberger Equation (KSE) [18], which constitutes a blueprint for the construction of the proof automaton. This fixed point coincides with the solution of the optimization problem formulated in (23) (24), when it has a solution. The general form of KSE is:

$$Z = E(p) \cdot Z + T(p) \tag{31}$$

In (31),  $E$  is a square matrix, with each entry a rational form constructed from the basis of inference operators described above, and  $T$  is a vector of equational forms from the Knowledge Base. Each non-empty entry  $E_{i,j}$  in  $E$  corresponds to the edge in the proof automaton connecting states  $i$  and  $j$ . The binary operator “.” between  $E(p)$  and  $Z$  represents the “apply inference to” operator. Terminal states are determined by the non-empty terms of  $T$ . The  $p$  terms are custom parameter values in the inference operator terms in  $E(\cdot)$ .

A summary of the procedure executed by the inferencer is presented in Figure 11.

We note that the construction of the automaton is carried out from the canonical equation and not by a non-deterministic application of the inference rules. This approach reduces the computational complexity of the canonical equation (low polynomial) and is far better than applying the inference rules directly (exponential).

The automaton is simulated to generate instances of the state, action and evaluation variables using an automaton decomposition procedure [50] which requires  $n \log_2 n$  time, where  $n = \#$  of states of the automaton. This “divide and conquer” procedure implements the recursive decomposition of the automaton into a cascade of parallel unitary (one initial and one terminal state) automata, see Figure 12. Each of the resulting automata on this decomposition is executed independently of the others. The behavior of the resulting network of automata is identical with the behavior obtained from the original automaton, but with feasible time complexity.

In Figure 12, the Parallel Decomposition transformation decomposes the finite state machine into a parallel series of unitary automata. A unitary automaton is a finite state machine with a single initial state and a single terminal state. The Cascade Decomposition then transforms each unitary automaton

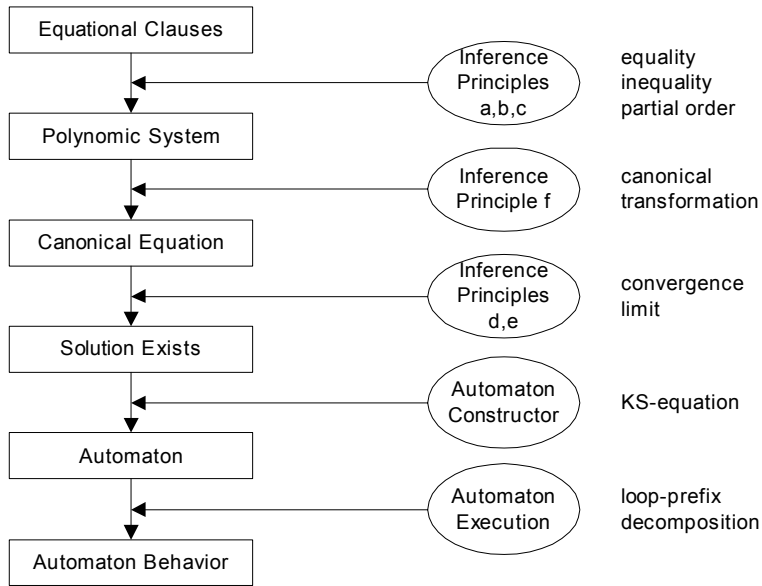


Figure 11: Summary of Inferencer Procedure

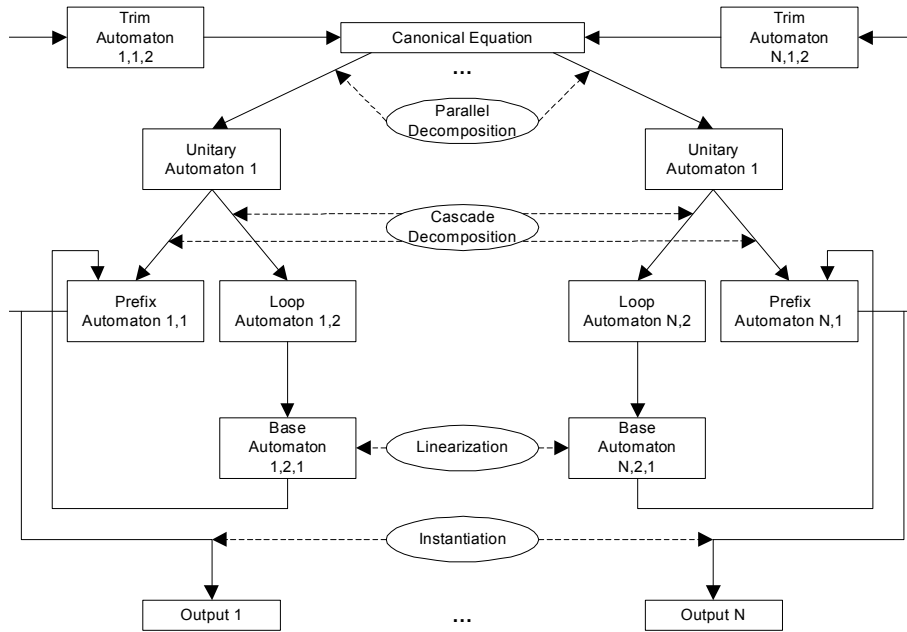


Figure 12: Automaton Decomposition Procedure

into a series of a prefix automaton followed by a loop automaton. A prefix automaton is a unitary automaton which has no transitions from its terminal state and a loop automaton is a unitary automaton whose initial state and terminal state coincide. The Linearization modifies the loop automaton by incorporating a new terminal state which has the same edges as the initial state of the loop automaton. Then the inaccessible states are trimmed from the resulting automata and the entire decomposition procedure is repeated if necessary. Whenever an automaton is produced in this decomposition which consists of a single path from the initial state to the terminal state, such an automaton is called a path automaton; it corresponds to a successful path in the original finite state machine and an output is produced.

The  $n \log_2 n$  complexity of the procedure described above is a direct result of the non-deterministic nature of the procedure. The partial results of executing each path through the procedure are collected in the corresponding output states pictured at the bottom of Figure 12. The first path automaton in the decomposition produced by the procedure is executed and the result is stored in the corresponding output state. Once a successful path automaton is produced and executed, the decomposition procedure is terminated (first\_finish\_halt\_strategy).

The inferencer for each Control Agent fulfills two functions:

- (i) to generate a proof for the system behavior theorem of each agent generated by the Planner (equations (20) and (21)) and
- (ii) to function as the central element in the Knowledge Decoder.

We now describe its function for proving the behavior theorem. Later, we will overview its function as part of the Knowledge Decoder. To show how the inferencer is used to prove the Planner theorem, (23), (24), first, we show how this theorem is transformed into a pattern of the form of (25). Since (23), (24) formulates a convex optimization problem, a necessary and sufficient condition for optimality is provided by the following dynamic programming formulation:

$$\begin{aligned}
 V_i(Y, \tau) &= \inf_{\alpha_i} \int_{\tau} L_i(\Psi_i(\tau, Y), v_i|_p(G_i(\tau, p))) d\alpha(p, d\tau) & (32) \\
 \frac{\partial V_i}{\partial \tau} &= \inf_{\alpha_i} H\left(Y, \frac{\partial V_i}{\partial \tau}, \alpha_i\right) \\
 Y(t) &= p \\
 \tau &\in [t, t + \Delta)
 \end{aligned}$$

In (32), the function  $V_i$ , called the optimal cost-to-go function, characterizes minimality starting from any arbitrary point inside the current interval. The second equation is the corresponding Hamilton-Jacobi-Bellman equation for the problem stated in (23) and (24) where  $H$  is the Hamiltonian of the relaxed problem. This formulation provides the formal coupling between deductive theorem proving and optimal control theory. The inferencer allows the real-time optimal solution of the formal control problem resulting in intelligent distributed

real-time control of the multiple-agent system. The central idea for inferring a solution to (32) is to expand the cost-to-go function  $V(\cdot, \cdot)$  in a rational power series  $V$  in the algebra:

$$\tilde{D}\langle\langle(Y, \tau)\rangle\rangle \quad (33)$$

Replacing  $V$  for  $V_j$  in the second equation in (32), gives two items: a set of polynomial equations for the coefficients of  $V$  and a partial order expression for representing the optimality. Because of convexity and rationality of  $V$ , the number of equations to characterize the coefficients of  $V$  is finite. The resulting string of conjunctions of coefficient equations and the optimality partial order expression are in the form of (25). A detailed algorithmic approach to solving (32) which we call hybrid dynamic programming can be found in [35].

In summary, for each agent, the inferencer operates according to the following procedure.

**Step 1:** Load current theorem (23), (24).

**Step 2:** Transform theorem to equational form (25) via (32).

**Step 3:** Execute proof according to figure 11.

If the theorem logically follows from the Knowledge Base (i.e., it is true), the Inferencer procedure will terminate on step 3 with actions. If the theorem does not logically follow from the Knowledge Base, the Adapter is activated, and the theorem is modified by the theorem Planner according to the strategy outlined above. This mechanism is the essence of reactivity in the agent. Because of relaxation and convexity, this mechanism ensures that the estimatable set of the domain is strictly larger than the mechanism without this correction strategy.

### 3.1.5 Knowledge Decoder

The function of the Knowledge Decoder is to translate knowledge data from the network into the agent's Knowledge Base by updating the inter-agent specification clauses. These clauses characterize the second constraint in (32). Specifically, they express the constraints imposed by the rest of the network on each agent. They also characterize the global-to-local transformations (see [30]). Finally, they provide the rules for building generalized multipliers for incorporating the inter-agent constraints into a complete unconstrained criterion, which is then used to build the cost-to-go function in the first expression in (32). A generalized multiplier is an operator that transforms a constraint into a potential term. This potential is then incorporated into the original Lagrangian of the agent which now accounts explicitly for the constraint.

The Knowledge Decoder has a built-in inferencer used to infer the structure of the multiplier and transformations by a procedure similar to the one described for (14). Specifically, the multiplier and transformations are expanded in a rational power series in the algebra defined in (33). Then the necessary conditions for duality are used to determine the conjunctions of equational forms and a partial order expression needed to construct a theorem of the form of (25) whose

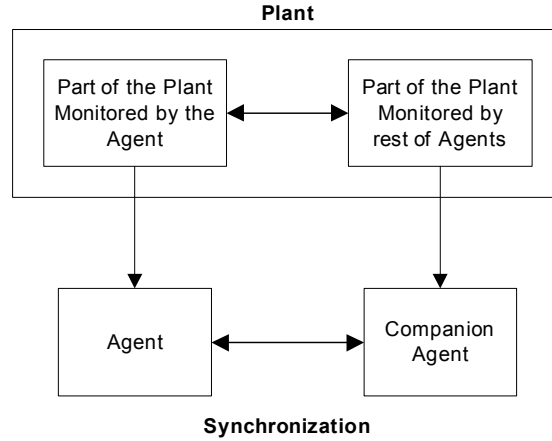


Figure 13: The Companion Agent

proof generates a multiplier for adjoining the constraint to the Lagrangian of the agent as another potential.

The conjunction of equational forms for each global-to-local transformation is constructed by applying the following invariant embedding principle:

For each agent, the actions at given time  $t$  in the current interval, as computed according to (32), are the same actions computed at  $t$  when the formulation is expanded to include the previous, current, and next intervals.

By transitivity and convexity of the criterion, the principle can be analytically extended to the entire horizon. The invariant embedding equation has the same structure as the dynamic programming equation given in (32), but with the global criterion and global Hamiltonians instead of the corresponding local ones.

The local-to-global transformations are obtained by inverting the global-to-local transformations, obtained by expressing the invariant embedding equation as an equational theorem of the form of (25). These inverses exist because of convexity of the relaxed Lagrangian and the rationality of the power series.

It is important at this point to interpret the functionality of the Knowledge Decoder of each agent in terms of what it does. The multiplier described above has the effect of aggregating the rest of the system and the other agents into an equivalent companion system and companion agent, respectively, as viewed by the current agent. This is illustrated in Figure 13.

The aggregation model (Figure 13) describes how each agent perceives the rest of the network. This unique feature allows us to characterize the scalability of the architecture in a unique manner. Namely in order to determine computational complexity of an application, we have only to consider the agent with

the highest complexity (i.e., the local agent with the most complex criterion) and its companion.

### 3.1.6 The Persistent Object Store

As described above, our Persistent Object Store System is the means of interfacing between an agent and the network controller and scheduler to implement the network of cooperating agents, see Figures 6 and 7. This system has a basis of generic simulation trajectory primitives that are instantiated during run time to values determined by simulation data and stitched together to form a trajectory corresponding to the behavior of the simulations. The goal of this system is to reduce the amount of state data that has to be transmitted between agents to construct the simulation trajectories.

Next we describe one of the key components in our approach: the use of light weight, high-performance data management as an enabling technology for DIS. DIS must balance computing, data management, and networking. High-performance computing has always played a central role in interactive simulations; on the other hand, the equally important companion role of high performance data management is only now emerging. High performance data management is important for DIS for several reasons.

1. Complex simulations produce large amounts of data, especially those running on high-performance computers. To minimize the total elapsed time of a simulation, it is important not only to reduce the processing requirements of the simulation, but also to reduce the time required to load data into the simulation, to store data produced by the simulation, and to move data between stages in the simulation. In other words, high performance data management can be equally effective as high performance computing at reducing the total elapsed time of a complex simulation.
2. The fact that a simulation is distributed provides the opportunity to optimize the simulation by not only moving the computational tasks to the data, as is traditional in simulations, but also, when appropriate, of moving data, or replicated data, to the tasks. More generally, distributed high performance data management provides the means to move the task to the data, the data to the task, or to use some hybrid strategy.
3. The specific goal of achieving a single integrated image of, say, a battlefield involving synthetic environments and realistic modeling and rendering of physical environments requires that large amounts of data be managed and distributed. High performance data management provides precisely the tools to do this.

Traditionally, scientific computing has faced a no-win situation: either develop and maintain a proprietary system for data management or use a general purpose database. In the past, the first approach could not exploit commercial software or standards, which the second approach could not provide the

performance demanded by most simulations. Fortunately, recent advances in operating systems, programming languages, and compilers has provided a third approach: develop light weight software tools for high performance data management. By light weight, we mean that a limited functionality is provided, but with the benefit that the software tool itself can be relatively simple. The result is high performance, with less effort required for porting and maintenance.

Another key idea is to replace file-based access to data with object-based access. With object-based data access, applications such as DIS Can easily exploit specialized application-specific algorithms for querying, storing and accessing data in order to obtain greater functionality and gain higher performance. In other words, rather than accessing unstructured data using the file system, one accesses the relevant structured objects by querying a persistent object store. Although the tables in a relational database are special types of objects, the type of scientific data occurring in simulations is typically of sufficient complexity that is usually more efficient to store and access it as complex objects, together with the appropriate pointers, than in the normalized form typical of relational databases. With both the data and processors distributed, agents perform a crucial function by connecting a request (or query) to the appropriate data and the appropriate resources.

A distributed interactive simulation may be thought of as a hybrid system consisting of a distributed collection of dynamical systems and automata.

1. The dynamical systems with inputs and outputs model the physical components of the simulation, the rendering of the environment, and other phenomena modeled by continuous laws.
2. The automata with inputs and outputs model the software components of the simulation and select the appropriate dynamical systems for a particular time period.

An agent is a collection of variables, predicates and rules; reference the previous description of the role of agents which manage processing and data requests. The hybrid systems which, by updating variables and evaluating rules and predicates, selects an automaton in the hybrid system to be replaced by a new automaton. That is, just as an automaton, which is internal to a hybrid system, selects an appropriate dynamical system, an agent, which is external to a hybrid system, selects an appropriate automaton. To summarize, we envision a distributed interactive simulation to also consist of a distributed collection of agents: The agents are rule-based systems which coordinate the different distributed components of the simulation by handling requests, monitoring the system components, and selecting appropriate automata to achieve a requested action.

Both automata and agents can be used to achieve a given behavior of a complex system by selecting one component from a collection of components. The differences are that automata operate on a fast clock by accepting an input symbol, transitioning to a new state, and outputting a symbol which specifies the appropriate dynamical system, while agents operate on a slow clock by

updating internal variables, recomputing predicates using rules, and selecting the appropriate actions determined by the rules.

Hybrid systems may be viewed from two complementary viewpoints: the discrete viewpoint in which the continuous system is viewed as a discrete collection of continuous objects, such as trajectory segments; and the continuous viewpoint, in which the discrete automaton is viewed as a continuous system with discrete transitions. From the discrete viewpoint, persistent object managers appear, for example, as a mechanism to manage physical collections of trajectories' objects. Once managed in this framework, it is a simple task to interface discrete digital automata to select trajectory segments with desired performance specifications or to provide appropriate control laws to the non-linear input-output system, see [11] for further discussion. An example of this type of interaction is found in [13] which describes a path planning algorithm for hybrid systems. In this algorithm during preprocessing, large numbers of trajectory segments for a hybrid system are computed and used to populate a persistent object store of trajectory segments. Subsequently, a request to approximate a desired flight path is interpreted as a query on the persistent store of trajectory segments. The query returns a sequence of trajectory segments which approximate the desired flight path. Near real-time performance results from using appropriate indices. One strength of this approach is that the actual controls required can be retrieved along with the trajectory segments at no additional cost.

Another application of persistent object stores is to use persistent object store of predicates to implement certain function of the agent. That is, an agent also has discrete symbolic data. It is natural to use a persistent object to manage this data. More specifically, a low overhead, high performance object manager can be used to manage the distributed collection of predicates required by a system of cooperating autonomous agents in order to provide the performance required for real time applications. High performance inferencing could then be achieved by the use of a companion software tool which would access predicate rules and facts stored by the persistent object manager and provide high performance inferencing. Thus a "light weight" inferencer plus a distributed, persistent object manager can provide an alternative to Prolog or LISP for implementing distributed agents.

Consider a DIS which uses multiple agents for the control of hybrid systems. The present implementation described in [33] for autonomous controlling agents uses compiled MetaProlog (Quintus Prolog) programs for each agent. The agents observe their local plant behavior and occasionally change the finite automaton controller of the plant when the performance specification is not met. With currently available commercial implementations, MetaProlog has serious known limitations when used for real-time control of large systems. There are two causes. One is the basic Prolog mechanism of backwards search. This one can be eliminated at some additional overhead by forward chaining implementations. But the other, more troublesome, cause is the necessity of retrieving and depositing rules and facts in large databases for on-line theorem proving in real-time. In the approach described in [11], the agent proves the theorem for a

discretized optimization problem expressed in automaton equation terms that there is a control law that is sufficiently optimal for use for the next interval of time.

The crucial element here is to eliminate the bottleneck due to having to consult a large database to prove this theorem, that is, to choose a control law that is essentially obtained by back propagation (backwards chaining) from the final goal. These databases of rules and facts can be very large because the dimensions and sizes of the state space of the controlled systems can be large. Prolog based systems are not very fast in performing this task, as the lack of commercial exploitation of deductive database systems over a twenty-five year period attests. General purpose databases and MetaProlog currently carry overhead that makes such large real-time applications hard to implement, necessitating disabling much of the Prolog mechanisms and using ad hoc database management techniques. Using a light weight persistent object manager together with a light weight inferencer to manage predicate data, stored rules and facts, as proposed in [11] appears to be a promising approach.

## 4 Architectural Elements of a Declarative Control Network

In MAHCA we synchronize state trajectories so that their evolution is continuous with respect to the topology defined by the stored knowledge. The only interagent synchronization mechanism MAHCA employs to achieve distributed simulation control is the result of the flow of equational logic terms between agents which alter the functional interaction of the agents with its simulator. A group of agents can exhibit causally inconsistent behavior due to the order of generation of events by two agents. An example case may be when a tank and helicopter are on opposing forces and are being simulated by different simulations. Suppose the tank has engaged an opponent in a house and the helicopter has engaged the tank. Suppose now that the result of the helicopter engagement is that the tank is destroyed. Because of latency, it may happen that the user agent is not aware of the tank destruction and it sets a goal for destruction of the house. Thus we have a loss of consistent local times between the agent controlling the helicopter simulation and the agent controlling the tank simulation. This is manifested by a phase incoherence, or discontinuity, between the phase of the state behavior of the user agents and of the simulation agents. As soon as this is detected by the user agent inferencer, this detection causes a logic constraint clause to fail. A side-effect of this logic failure is the identification of the offending terms in the user agent plan. Reactivity, which includes an interagent request, results in resolving the logic failure. We refer to this sequence of steps as restoring phase coherence or synchrony.

The inter-agent communication network's main function is to transfer inter-agent constraints among agents according to a protocol written in equational Horn clause language. These constraints include application dependent data and,

most importantly, inter-agent synchronization. The inter-agent synchronization strategy is very simple. An agent is synchronous with respect to the network if its inter-agent constraint multiplier is continuous with respect to the current instance of its active knowledge. Since the equational Horn clause format allows for the effective test of continuity (which is implicitly carried out by the inferencer in the Knowledge Decoder), a failure in the Knowledge Decoder theorem results in a corrective action toward re-establishing continuity with respect to the topology defined by the current instance of the knowledge base [23].

The specification of the geometry of the network, as a function of time, is dictated primarily by global observability. By global observability, we mean closure of the knowledge of the system as whole relative to the scope the systems reactivity. One of the central tasks in any application is to provide knowledge in the equational clause format to characterize global observability for the hybrid systems.

#### **4.1 The Challenge of Communicating Systems of Equations between Autonomous Objects**

One of the key problems for any battlefield DIS systems is to minimize the information needed to communicate to give the location of objects such as a tank, helicopters, etc.

The MAHCA DIS architecture consists of cooperating demand agents and simulation agents which add intelligence to the current DIS architecture, providing the ability to reconcile differences among its component constructive, live and virtual simulations. In this context, the intelligent controllers sit between the physical communications network and the simulation nodes, as shown in Figure 1. They perform intelligent interpretation of the contents of all Protocol Data Units (PDUs) sent and received. PDUs often must include analog models transmitted as sets of equations.

The overall principle to minimizing the communications bandwidth used to transmit information on the locations of simulation objects is that only state change information should be transmitted. Whenever a simulation object begins an activity which will continue for quite a while (where the definition of “quite a while” depends upon the viewpoint of the user of the simulation), the idea is to broadcast that the activity is beginning, and then to provide occasional updates. For example, conceptually one wants to transmit:

Event: start movement from A to B; and

Update: continuing movement from location C.

What is required is a type of “dead reckoning” strategy. That is, each node must maintain a model of its own objects that corresponds to the model(s) being used by all the other nodes, and that it must continuously compare its current information with the approximations being used by the other nodes. Thus when a state update is transmitted, we must include not only the correct position and orientation but also the velocity vectors and other information that is required

for the other node to compute a new approximation of the object's position. Thus dead reckoning strategies correspond to equations that model continuous motion through time and space. For example, suppose a tank platoon sets off down a road at 40 mph. The most obvious correction to the model that the tanks are moving in a straight line is to recognize that they are following a road, and roads are rarely straight. A more sophisticated correction would be to notice that there are huge craters in the road at various points, and the tanks must slow down to get past those obstacles. Corrections for logical events, like encountering craters, are probably best handled by the tank movement simulator, which in some cases may broadcast an immediate notification of the interruption, and in other cases will provide implicit notification when the next periodic update to the tank column's actual location is broadcast. If dead reckoning were the only continuous process in time and space which had to be simulated, and if there were one best dead reckoning strategy, then the special case solutions which have already been implemented would be adequate. However, a variety of dead reckoning strategies will be needed to adequately model the motion of various kinds of objects (motion of ships and aircraft is more accurately handled by these methods). Moreover, dead reckoning is just one of a very large set of processes which are best modeled by sets of equations which describe relationships in time and space.

Hybrid systems theory provides both a conceptual basis and a computational framework for describing and implementing systems which are best described by a combination of evolution equations and discrete logical events. Dead reckoning is just one of the processes which is best described by a combination of logical and evolution models that our proposed MAHCA DIS architecture is designed to integrate with other simulation models.

Transmitting even a single equation between heterogeneous computers can be a challenging task. There are software engineering issues related to standardization of notation, compilation versus interpretation, and the accuracy of floating point representations of real numbers. There are daunting configuration management issues in a very large distributed system—the trust that the particular versions running at a given moment are consistent, for example. The declarative control architecture of MAHCA is the result of a fresh look at these problems and issues.

Consider the problem of providing timely updates of operational information to a joint task force commander of a light task force enroute from the Continental United States (CONUS) to insertion in an unsecured objective area. Suppose that the possible alternative courses of action for the joint task force depend upon alternative methods of crossing a water barrier currently spanned by a bridge. Then

- The operational scenario being executed by the task force commander will have been developed, shared, and updated by referring to military control symbols superimposed onto a map of the terrain in the area of operations as a set of overlays (e.g. objectives, unit boundaries, routes of advance, coordination measures, fire, communications, and medical support, barrier

plan, transportation plan, . . . );

- Depending on the time and tools available and the size of the force, the operational scenario may have been rehearsed with staff and operational elements using sand tables, map exercises, and Advanced Distributed Simulations. Information pertaining to alternative courses of action will have been developed during preparation for the operation and will have been stored in a variety of formats at different levels of command;
- Information available from commercial and military sources provides timely updates of weather information which, in turn, affects the trafficability of terrain, feasibility of use of various avenues of approach, and combat effectiveness of opposing forces;
- Information available from national technical means provides timely updates of opposing force size and disposition; and
- The task force commander and his staff need the current information on weather conditions, accurate maps of the operational area, updates of recent changes to terrain features which might not be reflected in the maps, and the dispositions of opposing and friendly forces to execute the task force operations plan upon arrival in the area of operations. The information currency and accuracy is critical, and it must result from the best electronic and human intelligence gathering means available (e.g., the latest satellite pass). This means that updates to the task force databases must occur before, while, and after the task force is in transit to the area of operations.

The problem of determining whether the bridge over a water obstacle is usable for the contemplated operation, and further, informing the joint task force commander and his staff in a timely manner exemplifies the issues in maintaining consistent views of the battlefield. The usability of the bridge for light armor operations depends not only on its ability to carry the heaviest piece of task force equipment but also on the trafficability of the avenues of approach to the bridge and avenues of egress from the bridge. Fire support plans and barrier plans for operations around the bridge are based on known enemy dispositions and anticipated movements. Air defense plans are developed based on air avenues of approach into the area of operations with plans to provide point defense of the bridge during task force operations around the bridge. Task organizations are designed to overmatch enemy ground and air capabilities in the area of operations. Logistics support plans (transportation, medical and personnel) are based on availability of the bridge for combat service support operations. Terrain and operational entity data change at different rates in different formats with different degrees of accuracy (trust). Data pertaining to the use of the bridge by various task force elements will come at different rates from multiple sources (reports from coalition partners, updates from multi-spectral satellite imagery, sensor readings from temperature, pressure, acoustic, magnetic and

other environmental sensors, reports from friendly forces, . . .). Commanders at different echelons need different levels of detail of local conditions at different update rates in order to create and execute orders for synchronization of combat power.

Now consider the problem of using automation-aided operations to assist in the operational rehearsal (and training) of force components in a virtual environment that generates the most likely battlefield situation/scenario; and actual operational execution, with command and control oversight, that is able to quickly adjust mission plans to changing situations.

Let's begin with the observation that if the communications infrastructure is good enough for an Advanced Digital Simulation (ADS) which meets DIS requirements, then it is certainly good enough to model communications on the battlefield. To keep things simple, let's assume that ADS communications have adequate bandwidth and no perceivable delay. This assumption is consistent with the DIS vision principles of "ground (absolute) truth" and autonomy of simulation nodes. The problem we will discuss in this section is how to model the low bandwidth, queuing delays and service interruptions of actual battlefield communications in the ADS context. To make the discussion even more concrete, suppose we are focusing on receipt and transmission of digitized messages by a particular tank. At the top of Figure 14 there is an idealized Tank Simulator which has a variety of inputs including static state information, location information (probably from GPS), (synthesized) imagery and tactical radio supporting both voice and data. Outputs of such a tank simulator might include location, time, orientation, velocity vector, specific elements, and tactical radio. Focusing specifically on the tactical radio inputs and outputs, there will be a variety of communications partners, and the existence and quality of the communications links to each of those partners will depend on both the overall environment (e.g., the overall level of electro-magnetic activity in the region) and specific factors (objects breaking the line of sight between two partners, antenna directionality, etc.). The remainder of Figure 14 shows one possible architecture for inserting tactical communications into the DIS environment. In the center of the diagram is a tactical communications simulator inserted as a simulation object. By the principle of autonomy, the tank and all its communications partners can see all communications activity. It is up to the PDU interfaces to those simulators to ignore all "ground truth" PDUs and to only see PDUs from the communications simulator. But this approach seems to violate the principle of autonomy of simulators because we have elevated the tank's *view* of tactical communications to the role of ground truth. This line of argument suggests that tactical communications must be treated in a similar manner to dead reckoning, with compatible models implemented in the PDU interfaces of every simulator.

However actual communications is orders of magnitude more complicated than dead reckoning. Individual simulators will model tactical communications at widely varying levels of abstraction, so the PDU interfaces must make appropriate translations. Levels of abstraction have proven useful for aggregation but a solution for disaggregation has yet to be implemented. For example, the

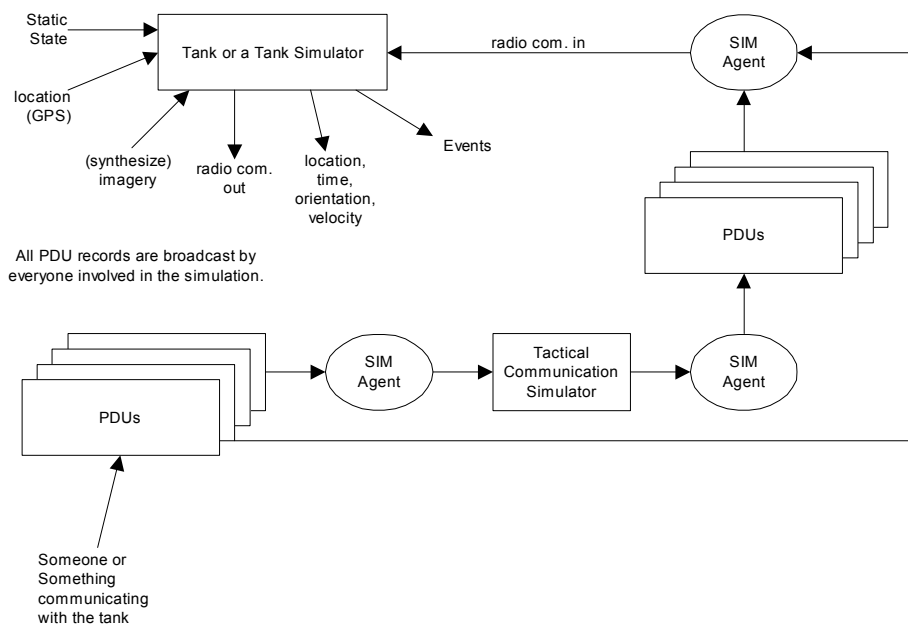


Figure 14: Possible (but inadequate) Simulation Architecture for Tactical Communications

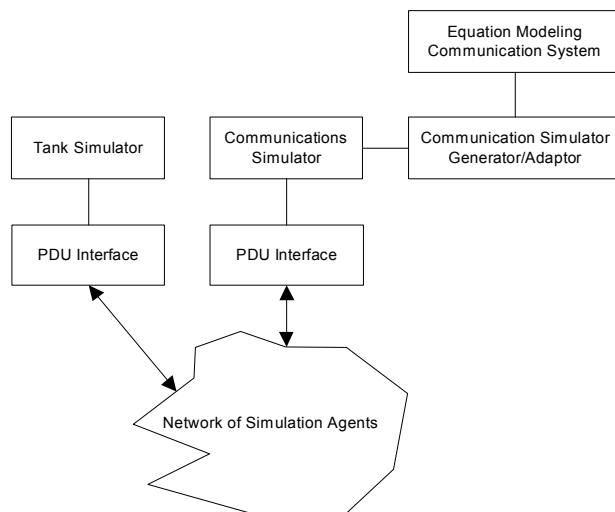


Figure 15: Declarative Control Architecture Approach to PDU Development

tank simulator may want to receive the actual ASCII string to be displayed to the tank commander, but the real system may depend on the transmission of the same information in an encoded binary string sent using a certain secure radio protocol. The actual load on the communications net will depend on the encoding scheme, the protocol, and the communications media. The delivery time or failure to deliver will depend on priority, other traffic in the environment, and the positions of sender and receiver. While aggregation of message traffic at higher headquarters into sets of ASCII strings is useful for many purposes, disaggregation of the sets of strings into appropriate sets of the lower level forms (which determine operational loading of the communications network) is a very hard problem. This discussion leads to the model in Figure 15. The PDU interface to the tank simulator needs to share information with its simulation agent which tells it how the environment affects its ability to perceive radio signals in its environment. Such an agent could either provide a special entry to a special-purpose communications simulator running in the ADS environment, or it could provide a copy of a (standard) communications simulator which is integrated into the tank simulator's PDU interface.

In summary, MAHCA provides heretofore unavailable capabilities to the DIS community. Because models are "wrapped" with agents which implement formal analytical models of the simulation, new constraints and capabilities can be incorporated quickly. Because the executable simulation code is generated from the formal models, it is possible to meet efficiency requirements with code that is known to be correct. Because the MAHCA has already been demonstrated on similar problems, the approach is known to be practical. Because the approach makes full use of existing models, a powerful capability can be brought on-line

quickly and the vision of realistic, shared, synthetic theaters of war will be much closer to fruition.

## 5 Conclusions

Our formulation gives a precise statement of the DIS communication network control problem in terms of a multiple agent hybrid control architecture. Our approach characterizes the problem via a knowledge base of equational rules that describe the dynamics, constraints and requirements of the DIS communications network (channels, switching nodes, customer characteristics, scheduling and planning strategies, etc.).

We have developed a canonical representation of interacting networks of controllers. Given a connectivity graph with  $N$  nodes (controllers) and the corresponding agent's knowledge bases, a network of  $2N$  agents can be constructed with the same input-output characteristics, so that each agent interacts only with another (equivalent) companion agent, whose knowledge base is an abstraction of the knowledge in the network. Thus, in general, the multiple-agent controller for any network configuration is reduced to a set of agent pairs.

One agent of the agent pair maintains coordination with other agent pairs across the network. We call that agent of the pair which represents network information the Theveninn Agent, after the author of a similar theorem in electrical network theory. The proof carried out by the Theveninn Agent generates, as a side effect, coordination rules that define what and how often to communicate with other agents. These rules also define what the controller needs from the network to maintain intelligent control of its physical plant.

Our approach develops a canonical way to prove the theorem characterizing the desired behavior for each agent by constructing and executing on-line a finite state machine called the "proof automaton." The inference process is represented as a recursive variational problem in which the criterion is an integral of a function called the Generalized Lagrangian or Demand functions. The Generalized Lagrangian maps the Cartesian product of equational rules and inference principles to the real line, thus effectively providing a hill-climbing heuristic for the inference strategy of the theorem prover (see section 3). In MAHCA the inference steps play a role analogous to action signals in conventional control, which vector fields on the manifold  $M$  constitute generators of feedback laws.

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