A New Approach to Generating Finite-State Control Programs for Hybrid Systems

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Abstract

We provide an abstract of a new approach for generating finite-state control programs for hybrid systems. We illustrate these ideas in the context of a cost-based Computer-Aided Control Engineering (CACE) environment. We discuss our methodology for ensuring that the environment should support automatic generation of automata which simultaneously comply with logic and evolution constraints. We argue that a declarative, hybrid systems approach to off-line design and on-line generation of reactive control automata is needed for achieving synchronization, scalability, integration, and incremental construction of large-scale, computer-controlled systems. The methodology supports a multiple-game strategy for distributed systems optimization whereby different components of the systems can employ different local optimization strategies. We conclude by comparing this constructive approach for on-line selection of actions that are near-optimal with respect to economic cost to the simulation-based approach for heuristic and exhaustive trade-off analysis.

Keywords: near-optimal, hybrid, nonlinear, intelligent, automata generation, distributed control, reactive synchronization

1 Introduction

Recent results of a joint IFAC-IEEE study on challenges of Computer Science in industrial application of control emphasize the need for improvements in computer-aided control engineering (CACE) cools, including investigating use of hybrid systems theory [1]. Hybrid systems are those systems described by compositions of logic models and evolution models. In this paper we provide a brief introduction to recent results in establishing mathematical foundations for hybrid systems which are appropriate for building improved tools for implementation of computer-controlled machines.

Current CACE environments provide dramatically improved methods for entering both logic and evolution models of complex systems and conducting simulation experiments with closed-loop and open-loop system behavior. In addition, some generate control programs based on those experiments. However, while this approach works well for small-scale systems, the experimental approach for integration of logic and evolution models is a major source of increased costs and decreased reliability for large-scale systems. The importance of achieving a synthesis capability for design and implementation of computer-controlled machines was underscored by Dr. John Cassidy in his plenary talk at the 1993 American Control Conference (ACC) when he said "*... 77% of the software of control systems is for implementation of logic and scheduling and 23% of the software is* *for implementation of control algorithms. We still do not have a methodology for integrating logic and control algorithms.*"[1] He emphasized the importance of creating such a methodology for integration of logic and scheduling software with control programs, the former based on set-based techniques and the latter based on differential equation theory, control systems theory and analysis techniques.

We discuss the Kohn-Nerode multiple-agent hybrid control architecture (MAHCA) as a means of overcoming the technical shortfall described by Dr. Cassidy. MAHCA is based on extensions to the theory of *relaxed variational control*. The basic theory was developed by L.C. Young in the 1970's [9] but implementation of the approach has been computationally infeasible. New results support computational feasibility of relaxed variational control [**?**, **?**]. This new theory extends the concepts, principles and algorithms of declarative control theory and merges them with principles of concurrent computing and dynamical hybrid systems in a formal framework based on variational models. The result is an optimization-based approach for achieving near-optimal, closed-loop performance of systems described by compositions of logic models and evolution models. The fundamental result is a constructive algorithm for realization, at each update interval, of an executable automaton which simultaneously complies with logic and evolution constraints.

Other research efforts are ongoing concerning development of alternative approaches for analyzing, designing, and implementing hybrid systems (see [**?**, **?**] for references). Advances in these areas would support creation of analogous hybrid systems CACE environments.

[1] Dr. John Cassidy, Director of Research, United Technologies Corporation, Plenary Speech, Control Technology and the 21st Century, American Control Conference, San Francisco, CA, 2 June 1993.

2 Implementing a Cost-Based Approach to Control Implementation

Research software has been used for the past three years to create single-agent and multiple-agent demonstrations of the hybrid systems approach to synchronization of distributed processes. However, the multidisciplinary tools do not currently exist.

2.1 Advantages of the Cost-Based Approach

Applying the MAHCA technology to overcome shortfalls in the current approach (fixed-task decomposition and synchronization through experimentation) would lead to the following imprevements in evaluation of control implementation alternatives:

- Unanticipated instabilities encountered at lower layers due to time lag can be automatically compensated,
- Logical inconsistencies occurring across distributed agents that are difficult to detect and repair can be detected and corrected,
- Synchronization of distributed processes currently achieved through experimentation can be synchronized based upon satisfaction of relaxed constraints,
- Specifying interoperability of components can be simplified by achieving support for incrementally declaring interoperability constraints,
- Scalability to large distributed systems can be supported through addition of agents in the logic communications network, and
- System continuity for distributed processes can be maintained through reactive synchronization of actions by the network of agents.

2.2 Generation of Programs (automata) using the Cost-Based Approach

The MAHCA framework described above provides a mathematically sound and complete cost-based approach for (1) evaluating different configurations of major components based upon alternative separation of the problem into different operations performed at different time scales at different locations at the system design stage and (2) performing on-line modification of the architecture in accordance with system goals, costs associated with the goals, and benefits of achieving the goals at the implementation stage.

The basic advantage that MAHCA has over alternative approaches is thet automatic generation of programs which comply with the set of logic and evolution constraints. Space limitations preclude a more complete description of the technology but two crucial steps are stating the problem as a cost-based, relaxed variational form and then solving this problem statement in terms of a finite number of actions which provide a near-optimal solution of the cost-based constraints.

These two steps require consideration of the state of the system as occurring on a differentiable manifold, which Kohn calls the carrier manifold of the system. A recent result [7] applies a Finsler metric ground form te efficiently calculate connection coefficients for a moving frame along trajectories occurring in the carrier manifold. Relevant notions are vector fields, derivations, and flows on manifolds.

For this paper, the significance of this result is that is provides a symbolic calculus for parallel transport from the goal state to the current state, and this parallel transport solution is then used to compute cost-benefit policies to move from the current state to the goal state.

2.2.1 Bellman's optimality principle

A state-space oriented assertion of Bellman's optimality principle is that an optimal polity has the property that whatever the initial state and the initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. For hybrid systems, application of this principle leads to the Hamilton, Jacobi, Bellman (HJB) relaxed variational form [7]:

$$
\begin{aligned}
V_i(Y,\tau) &= \inf_{\alpha_i} \int_\tau L_i(\Psi_i(\tau,Y), v_i\big|_p(G_i(\tau,p)))\, d\alpha_i(p,d\tau) \\
\frac{\partial V_i}{\partial \tau} &= \inf_{\alpha_i} H\big(Y, \frac{\partial V_i}{\partial Y}, \alpha_i\big) \qquad (1) \\
Y(t) &= p \\
\tau &\in [t, t+\Delta)
\end{aligned}
$$

Where p is a point in the carrier manifold, V_i are tangent vectors at each point and a_i are coefficients of the tangent vectors.

2.2.2 Optimality result

Here we summarize the main elements of our optimality results for the problem described by (1). The details are presented in the sections below.

- Control actions: The near-optimum control action at time t is given by a chattering combination of the basis of infinitesimal control actions. This basis is made by the basis vectors of the tangent space to M at the current state point p. The coefficients of the chattering are determined by solving a parametric linear programming problem (see [**?**]).
- State trajectory: the feasible behavior of the system is a flow on the manifold from the current point to the goal set. Each trajectory of this flow is a geodesic determined by an affine connection in the carrier manifold.
- The affine connection in M: This connection is determined by Christoffel symbols associated with the metric tensor given by the Hessian with respect to the rate of the Lagrangian function in (1). Sometimes this connection is referred to as the Finsler connection (see [7]).
- The parametric linear program for determining the chattering combination for computing the infinitesimal control action at p is formulated as follows: Parallel transport back the goal

transversal field along a geodesic to the current local goal point: that is, the point in the manifold corresponding to $t+dt$. Then find the chattering combination of basis infinitesimal control actions at time t so that the resulting control vector field is parallel to the goal field at time $t+dt$.

The appropriate interpretation of this solution is that the coefficients of the tangent vectors satisfy the Levy Civita continuation equation (affine connection) from x to $x+\Delta x$ and coincides with the HJB equation from x to $x+\Delta x$.

The revolutionary consequence of this result is that the Bellman Optimality Principle is the consequence of continuation in the hybrid systems formulation. The effect is that instead of having the previous limited result for linear systems that *if a solution exists*, then the Linear Quadratic or H^∞ result is exact, we can now use infinitesimal actions to both define the choices available, and then construct the HJB equation to be solved, secure in the knowledge that the T0 topology of hybrid systems tends to the Hausdorf topology in the limit. The result is summarized below:

We note that both actions and disturbances are transformations on points in the carrier manifold. Each infinitesimal action or disturbance is represented as a derivation on the manifold M as discussed above. We construct an inference automaton which is the epsilon-optimal hybrid controller (Δ approximation). The evolution equations are:

$$\begin{aligned} q_{t+\Delta} &= \delta(q_t, \omega_{t,t+\Delta}) && \text{neighborhood transition} \\ \omega_{t,t+\Delta} & && \text{solution of current equational terms} \end{aligned} \tag{2}$$

$$\begin{aligned} v\big|_q(\,) &= \beta(q_t) && \text{infinitesimal action} \\ u_{t+\Delta} &= \exp(\Delta \cdot v\big|_q)(H(X,g)) && \text{current control law} \end{aligned}$$

The logic equations are:

$$\begin{aligned} Y_t &= E(q_t)\cdot Y_t + K(q_t) && \text{Kleene-Schutzenberger Equation} \\ & && \text{Resolution of current relations} \\ \omega_{t,t+\Delta} &= S(Y_t) && \text{Selector function} \end{aligned} \tag{3}$$

Following the discussion of [6], how does one compute with the connection along an integral curve on manifold M? Suppose we are given

$$\alpha : [t_0, t_1] \rightarrow M \tag{4}$$

Suppose that x_0 is the initial point and x_1 is the endpoint of the curve, i.e., assume that $x_0 = \alpha(t_0)$ and $x_1 = \alpha(t_1)$. Suppose that curve lies entirely within some chart (U, Ψ) of the manifold, where U is an open set of the manifold and Ψ is a homeomorphism of U to a region of Euclidean space. Assume that the manifold is n-dimensional and that in local coordinates of the chart, the curve α is given by $\Psi(\alpha(y)) = (\alpha_1(t), \ldots, \alpha_n(t))$.

Let TM_x denote the tangent space of M at x. Now suppose that we are also given a tangent vector $y_1 \in TM_{x_1}$. We want to do a parallel transport of the tangent vector $y_1 \in TM_{x_1}$ at x_1 along α to x_0 to get the tangent vector y_0. This requires transporting x_1 as a vector of t, $g(t)$, tangent to the surface at $\alpha(t)$ for all intermedia t. But then $g(t)$ is in the tangent bundle of the manifold α. The condition that $g(t)$ is parallel along α is that the covariant derivative be zero along that path. Omitting reference to α, this says that $\frac{\partial}{\partial x_i}$ satisfies:

$$\begin{aligned} 0 &= \nabla g \\ &= \sum_i \left[\frac{dg_i}{dt}\frac{\partial}{\partial x_i} + g_i \nabla \frac{\partial}{\partial x_i} \right] \\ &= \sum_{ijk} \left[\frac{dg_k}{dt} + g_i \frac{d\alpha_j}{dt}\Gamma^k_{ij} \right] \frac{\partial}{\partial x_k} \end{aligned} \tag{5}$$

Hence $g(t)$ is parallel along $\alpha(t)$ if and only if the coordinate functions $g_i(t)$ satisfy the following system of ordinary differential equations.

$$\frac{dg_k}{dt} + \sum_{ik} g_i \frac{d\alpha_j}{dt}\Gamma^k_{ij} = 0, \qquad k = 1, \ldots, n. \tag{6}$$

Here the Γ^k_{ij} are the affine connection symbols. If the manifold is provided with a metric, these symbols are computed as the Christoffel symbels. In our variational problem, there is a natural metric that can be imposed that is compatible with the variational problem. The natural metric is the Hessian of the Lagrangian, L, with respect to $\dot{x}$. The Hessian constitutes a metric tensor because along optimal trajectories it is positive definite. This condition is the central defining property of Finsler manifolds.

3 Enabling Multiple Optimization Strategies in a Cost-Based Approach to Generation of Control Automata

Static *Actual Cost* analysis, widely used in industry [ ], is based on historical experience tables. All historically based analysis suffers because there are no strictly similar past processes. To compensate, analysis has to be based on untested and usually invisible subjective judgments. No dynamics of evolution for the processes budgeted are employed. A priori values are assigned at the beginning of the analysis are not usually updated in the light of experience. Statistical distributions are assumed. There is no basis for testing these assumptions. Bad decisions are revealed only after they have been implemented. There are no self-correcting mechanisms. No active structure is provided to represent how decision makers interact generating economic actions. Unlike dynamic models, *Actual Cost Analysis* cannot take into account non-linearities due to feedback between components, or even between economic variables. Finally, near-optimal plans and budgets for components cannot be constructed based on purely local inputs and outputs of that subbing. To achieve this goal one has to take into account interactions with other entities and global goals. This is beyond the capabilities of *actual cost* analysis schemes unless the latter is radically changed.

There have been, up to now, no Cost/Benefit software which incorporates heterogeneous organizational components which have different optimality criteria based on a variety of behavioral cost functions for different components. Even for large homogeneous organizations, current algorithms and models are not fast enough to update budgets and plans on-line.

Mathematical models and experience tables for planning and optimization by a given economic agent such as one of the sub-components above, are the standard stuff of operations research, management science, and applied economics. But how do we optimize and plan for economic organizations comprised of heterogeneous sub-components? Can we develop a common framework which will allow us to compute robust and near optimal plans and decentralized budgets to meet global organizational goals? These issues are partly addressed by the application of theoretical and applied economics by planning and decentralization. Computing has always been important to planning.

We propose to develop an architecture, based on hybrid systems theory and dynamic Games. In general a game is characterized by a collection of rules. When human agents are involved, interaction always involves some level of competition. The formulation of this interaction ranges from formal games to situations where rules are difficult to discern. Social scientists and economists both have endeavored to characterize significant cases of competition by abstractions which select rules and hence games. Many such games have been carefully studied mathematically, defining possible stategies and giving deep insight about many forms of human competition. We have taken the view that in the large, humans rarely apply "pure" strategies, but rather compete by combinations of strategies fashioned from unique points of view and understandings of the course of play. From this point of view, we have applied MAHCA in search of solutions which have not previously been recognized.

We believe that a recent paper [**?**] was the first to construct a principled mathematical model and algorithms for integrating the agents and their individual behaviors, their negotiations, their differing cost functions, and global organizational goals into a single model which can be used to compute in real time components plans and budgets which are cost effective relative to the behavior of the sub components, and assure, without centralized controls, that global goals and constraints are met.

The Multiple agent hybrid control architecture (MAHCA) for the first time allows the implementation of a heterogeneous set of dynamic, simultaneous game strategies. These games are played between multiple agents participation in economic policy modeling and game law extraction. The pomposite stategy coming from the application of the individual stategies, generates a minimum cost for implementation of design initiatives. This behavior is obtained in the presence of knowledge uncertainties, such as incomplete models of the systems being designed, loosely defined operational requirements, and evolving design parameters.

MAHCA will use a Cost/Benefit model in which the sub-components of a heterogeneous economic organization and their interactions constitute a partially decentralized "Cost/Benefit" economic game played between sub-components. The MAHCA model then becomes a dynamic economic game played between players with heterogeneous cost functions and with both logic and continuous evolution constraints. The budgets so obtained are "decentalized" plans for individual sub-components that meet material, financial, informational, and behavioral constraints on the heterogeneous organization. MAHCA algorithms assure that assuming the organizational structure given, the plans and budgets computed are robust and cost-effective or, using a more technical term, near optimal. Economic models with rules for interaction and heterogeneous competitive strategies have been studied for many years in economics for manufacturing, legislative, executive, enterprise, and other processes.

The central result of this paper is thath the composite strategy coordinating the agents of achieving a minimum cost is obtained by application of an extension of the Kohn-Nerode procedure for control law extraction to the near optimal solution of the dynamic cost analysis problem. A solution of this problem is characterized by the construction of game laws that satisfy multiple inter-agent strategies.

In the cost analysis application we discuss in this paper we assume that the agents play differential games. This is not a limitation because by our continualization procedure, any discrete game can be transformed into an ϵ-approximate differential game.

3.1 Problem formulation

We proceed to formulate the game problem we consider in this paper. Suppose that there are N players (herein referred to as agents) ($N > 1$). As a function of interdependency, which is determined by the application, we group the N players into not necessarily disjoint subsets $S_1, \ldots, S_k$. Within each subset, the players play a common game according to a set of rules pre specified for the subset. Notice that since the subsets S_i are not necossarily disjoint, a given agent may be playing in more than one game. This arrangement implies that the action of an agent has te simultaneously comply with game rules corresponding to more than one game. In conventional formulations this leads to inconsistencies. However, if we are willing to allow as a solution of our games a *two-time scale chattering solution* we obtain implementable game strategies for this problem. In the next paragraphs we will overview the nature of the solution which is the basis of our proposed cost analysis implementation.

An example of a game formulated cost analysis system of the type introduced above is shown in Figure 1. The system has three agents controlling the (cost) process. Each agent observes the process via its localized sensors and acts on it via its localized actions. In addition, the agents interact dynamically with each other via an interagent network. The three agents are playing two simultaneous games: S_1 involving agents 1, 2, and 3 and S_2 involving agents 2 and 3. They are supposed to conduct a game whose effects are the generation of actions that modify the behavior of the process. We will discuss the nature of these actions in a later section. For now we proceed to overview the main characteristic of our proposed solution.

In the two-time scale chattering solution each agents contends with two hierarchically organized time scales: the *game scale* and the *action scale*. This is illustrated in figure 3. Each game step takes D seconds. Each agent must determine what portion of this interval, $\Delta^i_{n_k} = \beta^i_{n_k}\Delta$, will be assigned to play according to n_k, one of a finite set of *basis games*. Then, within each subinterval $\delta^i_{n_k}$ the agent has to decide the time it assigns to l_s one of a finite set of

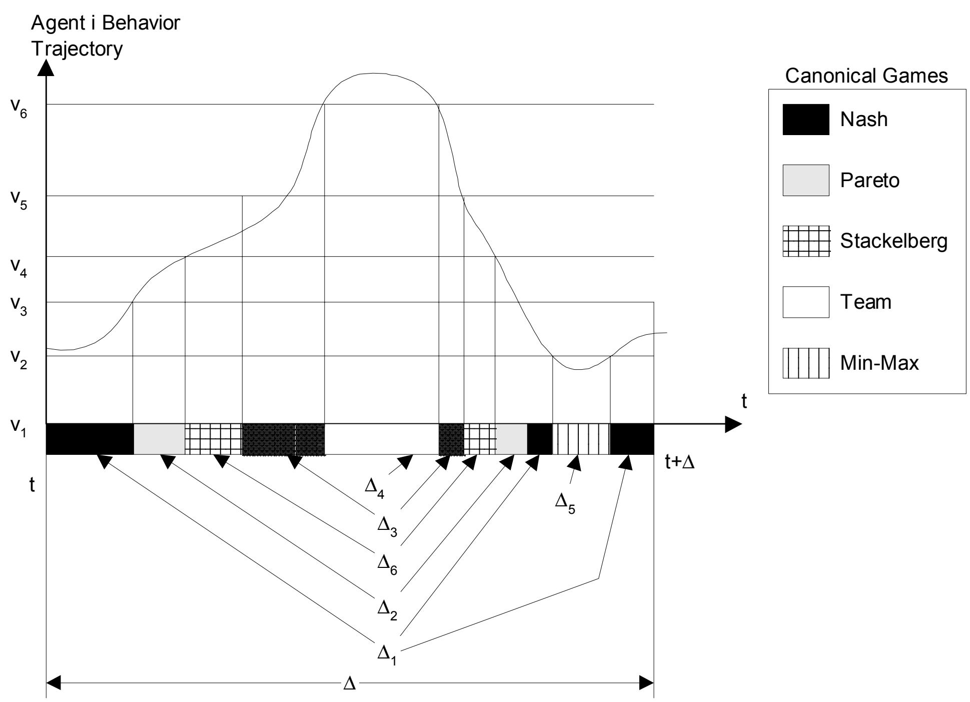

Figure 3: Basis Game Chattering

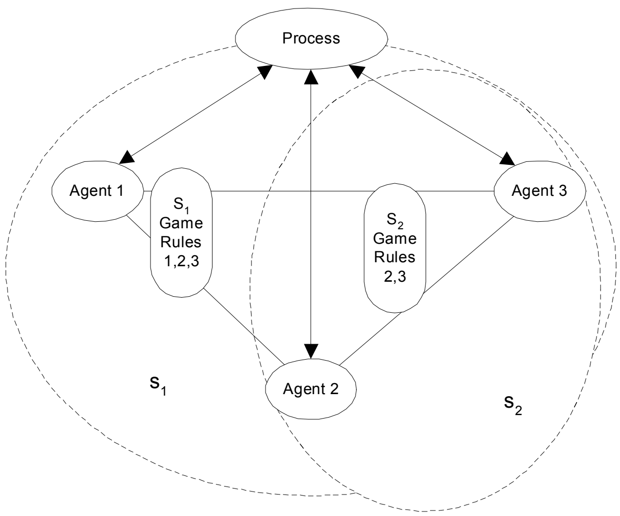

Figure 1: Example of Multiple Game arrangement

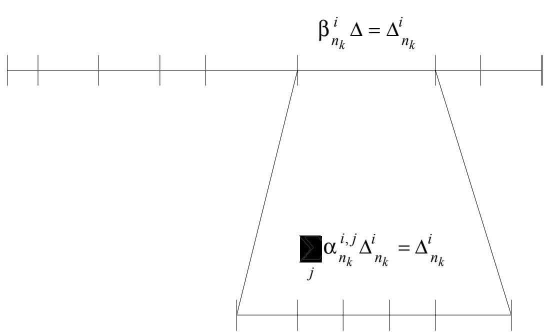

Figure 2: Two-time Chattering scale for agent i

pre-established *primitive infinitesimal actions* [**?**].

Each basis game is characterized by a different (rule) model. The heterogeneous game is carried out by implementing each game agent with its associated game and process rules as a MAHCA agent. As illustrated in figure 3, a MAHCA agent is performing on-line two hierarchically organized functions: 1- construction of a composite game strategy by game chattering combination and 2- Generation of infinitesimal actions, that are chattering combinations of the primitive *infinitesimal* actions available to the agent. The other function of MAHCA, adaptation [**?**] works similarly as the one used in control problems. Because of space considerations adaptation will not be discussed in this paper.

The idea behind our proposed solution of the game problem an agent is involved in is to construct an arbitrarily-close behavior (to the behavior of the game) by chattering among behaviors of suitably selected *basis* games. The solution of each basis game is obtained by chattering of game actions as in the control problem. This is illustrated in figure 2.

For the interested reader, Elie Cartan [3] was one of the first person to associate connections on Finsler manifolds with parametric variational problems. Parallel transport is discussed in [**?**]. Also, relaxed variational calculus is discussed in [9]. A more complete list of references can be found in [6].

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